GEOMETRY DAYS - TUIASI 2024

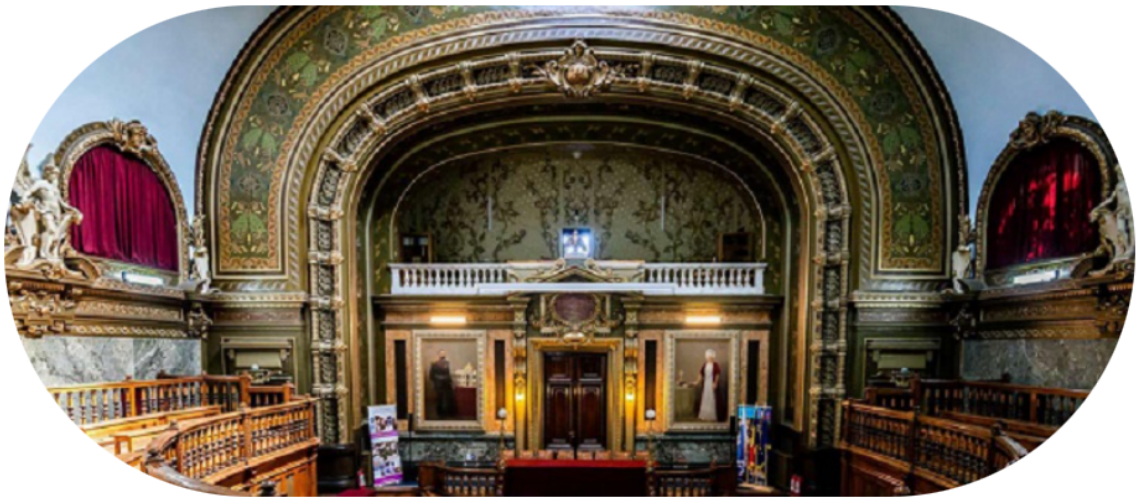
Department of Mathematics and Informatics, Gheorghe Asachi Technical University, Iasi, Romania

and

Nagoya Institute of Technology, Japan

JOINT WORKSHOP

6 - 9 September



BOOK OF ABSTRACTS

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1 About Geometry Days–TUIASI 2024

The objective of the workshop is to provide a forum and an opportunity of research interchanging for mathematicians in western and eastern in the area of differential geometry and its related fields.

Also, this workshop gives young scientists (including graduated students) an opportunity to get academic influence and to make their own presentations.

2 Plenary speakers PS

- **Toshiaki ADACHI** – Nagoya Institute of Technology, Nagoya, Japan
- **Naoya ANDO** – Kumamoto University, Kumamoto, Japan
- **Cornelia-Livia BEJAN** – Gheorghe Asachi Technical University, Iaşi, Romania
- **Ioan BUCATARU** – Alexandru Ioan Cuza University, Iaşi, Romania
- **Dorel FETCU** – Gheorghe Asachi Technical University, Iaşi, Romania
- **Hiroshi MATSUZOE** – Nagoya Institute of Technology, Nagoya, Japan
- **Marian Ioan MUNTEANU** – Alexandru Ioan Cuza University, Iaşi, Romania
- **Cezar ONICIUC** – Alexandru Ioan Cuza University, Iaşi, Romania
- **Gabriel Eduard VÎLCU** – Politehnica University and Gheorghe Mihoc – Caius Iacob Institute of Mathematical Statistics and Applied Mathematics of the Romanian Academy, Bucharest, Romania

3 Contributed speakers CS

- **Rareș-Mircea AMBROSIE** – Alexandru Ioan Cuza University, Iaşi, Romania
- **Ștefan ANDRONIC** – Alexandru Ioan Cuza University, Iaşi, Romania
- **Oana CONSTANTINESCU** – Alexandru Ioan Cuza University, Iaşi, Romania
- **Cristina Elena HREȚCANU** – Ștefan cel Mare University, Suceava, Romania
- **Aykut KAYHAN** – Maltepe University, Istanbul, Turkey
- **Takashi MATSUHISA** – MRI, BSBH Foundation Mito, Japan
- **Fumiko OHTSUKA** – Ibaraki University, Mito, Japan
- **Ioan Radu PETER** – Technical University of Cluj-Napoca, Romania
- **Keiko UOHASHI** – Tohoku Gakuin University, Sendai, Japan

4 Schedule

Day 1

8:30–9:00		**Registration**	
Chairman		**Hiroshi MATSUZOE**	
9:00–9:50	PS	**Dorel FETCU** Iași, Romania	Biharmonic and biconservative surfaces in Thurston geometries
9:50–10:00		**Coffee break**	
10:00–10:50	PS	**Toshiaki ADACHI** Nagoya, Japan	Trajectories on totally η-umbilic real hypersurfaces in a nonflat complex space form
10:50–11:20		**Coffee break**	
11:20–12:10	PS	**Marian Ioan MUNTEANU** Iași, Romania	Vector fields and magnetic maps
12:10–12:20		**Coffee break**	
12:20–12:45	CS	**Fumiko OHTSUKA** Mito, Japan	On the classification of non-degenerate regular polyhedral complexes
12:45–13:10	CS	**Takashi MATSUHISA** Mito, Japan	A geometric-algebraic interpretation of the generalized frequency of F. Milano after Y. Hasegawa and K. Horizumi
13:10—13:35	CS	**Aykut KAYHAN** Istanbul, Turkey	Biconservative hypersurface and differential equation
13:35		**Lunch**	

All presentations will be held in Room III.46

Day 2

Chairman	Toshiaki ADACHI		
09:00—09:50	PS	**Gabriel Eduard VÎLCU** București, Romania	Curvature invariants of submanifolds and stability of space forms
9:50–10:00	**Coffee break**		
10:00—10:50	PS	**Naoya ANDO** Kumamoto, Japan	Paracomplex structures and nilpotent structures
10:50–11:20	**Coffee break**		
11:20–12:10	PS	**Cornelia-Livia BEJAN** Iași, Romania	Codimension 2 submanifolds of paracosymplectic manifolds with normal Reeb vector field
12:10–12:20	**Coffee break**		
12:20–12:45	CS	**Keiko UOHASHI** Sendai, Japan	Comparisons of extended divergence on a foliation by deformed probability simplexes with other divergences
12:45–13:10	CS	**Rareș-Mircea AMBROSIE** Iași, Romania	Biharmonic homogeneous polynomial maps between spheres
13:10–13:35	CS	**Cristina Elena HREȚCANU** Suceava, Romania	Semi-invariant submanifolds of (α, p)-golden metric manifolds
13:35	**Lunch**		

All presentations will be held in Room III.46

Day 3

Chairman	Marcel ROMAN		
09:00—09:50	PS	**Hiroshi MATSUZOE** Nagoya, Japan	Constructing pre-contrast functions that induce specified Bartlett tensor fields
9:50–10:00	**Coffee break**		
10:00—10:50	PS	**Ioan BUCATARU** Iași, Romania	First integrals and symmetries in Finsler geometry
10:50–11:20	**Coffee break**		
11:20–12:10	PS	**Cezar ONICIUC** Iași, Romania	c-Biharmonic maps: a higher order conformally invariant generalization of harmonic maps
12:10–12:20	**Coffee break**		
12:20–12:45	CS	**Oana CONSTANTINESCU** Iași, Romania	Finsler metrizabilities and geodesic invariance
12:45–13:10	CS	**Ioan Radu PETER** Cluj-Napoca, Romania	Hardy type inequalities and curvature for nonreversible metrics
13:10–13:35	CS	**Ștefan ANDRONIC** Iași, Romania	Gap results for biharmonic submanifolds in Euclidean spheres
13:35	**Lunch**		

All presentations will be held in Room III.46

Day 1

September 6th, 9:00 AM – 13:35 PM

Biharmonic and Biconservative surfaces in Thurston Geometries

PS 9:00

Dorel FETCU

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We discuss old and new results on biharmonic and biconservative surfaces in Thurston Geometries, with an emphasis on the case when the ambient space is Sol_3.

Trajectories on totally η-umbilic real hypersurfaces in a nonflat complex space form

PS 10:00

Toshiaki ADACHI

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We study trajectories for constant multiples of fundamental forms on totally η-umbilic real hypersurfaces in a nonflat complex space form. These hypersurfaces are geodesic spheres, horospheres and tubes around totally geodesic complex hyperplanes. On a real hypersurface in a Kähler manifold, an *almost contact metric structure* is induced by the complex structure on the ambient space. The structure tensor field which corresponds to the complex structure on the ambient space induces the fundamental form. Important trajectories are *Legendre* ones, which are orthogonal to the characteristic vector field. Since Sasakian space forms, which can be regarded as odd-dimensional objects of complex space forms, are realized by some totally η-umbilic real hypersurfaces in complex space forms, we study trajectories by comparing them to those for constant multiples of Kähler forms on complex space forms. We show lengths of closed Legendre trajectories, study *pseudo-conjugacy* of Legendre magnetic flows, and give decomposed expressions of all trajectories on these real hypersurfaces. The key point is actions of *rotation factors*.

Vector fields and magnetic maps

PS 11:20

Marian Ioan MUNTEANU

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This talk is based on some joint papers with J. Inoguchi, Institute of Mathematics, University of Hokkaido, Japan. We recall here the recent work *Magnetic unit vector fields*, RACSAM 2023. In our paper [Magnetic maps, IJGMMP 2014] we define the notion of magnetic map as a generalization of both magnetic curves and harmonic maps. A magnetic map is obtained as critical points of the LH functional, that is the energy functional together with a potential part. As a vector field can be thought of as a map from the manifold to its tangent bundle and since the tangent bundle carries a natural magnetic field obtained from its almost Kaehlerian structure, we may ask when a vector field is a magnetic map? Furthermore, we show that a unit vector field on an oriented Riemannian manifold is a critical point of the Landau Hall functional if and only if it is a critical point of the Dirichlet energy functional. Therefore, we provide a characterization for a unit vector field to be a magnetic map into its unit tangent sphere bundle. Then, we classify all magnetic left invariant unit vector fields on 3-dimensional Lie groups.

On the classification of non-degenerate regular polyhedral complexes

CS 12:20

Fumiko OHTSUKA

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Our research object is a polyhedral complex which is an object obtained by gluing isometric edges of convex polygons. This is not necessarily a piecewise linear manifold, but a locally finite two-dimensional cell complex with a piecewise flat metric. A polyhedral complex is said to be regular if all its faces are congruent to the same regular polygon and the space of directions at each vertex is isometric to each other. The concept of curvature can be introduced via its vertex structure. Regular polygons and surfaces of Platonic solids are natural examples of positive regular polyhedral complexes. Also the objects obtained as their multi covering of them are so, but degenerate. In this talk, I would like to give an explicit definitions concerning our resarch object, and introduce many examples including those mentioned above. Then, I would like to show the classification theorem for non-degenerate positive regular polyhedral complex.

A geometric algebraic interpretation of the generalized frequency of F. Milano after Y. Hasegawa and K. Horizumi

CS 12:45

Takashi MATSUHISA

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This article presents the findings of Y. Hasegawa and K. Horizumi in their B. Eng. and M. Eng theses, submitted to Ibaraki University in 2024. The periodic phenomena represent a fundamental aspect of natural science. The investigation of periods, or equivalently that of frequency, has provided a new avenue of enquiry in modern science. Moreover, the investigation of extensions of frequency in diverse contexts has been a topic of significant interest in the field of engineering. F. Milano (2022) presented the concept of "generalised frequency" and provided a geometric interpretation from the perspective of classical differential geometry. Hasegawa and Horizumi initiated their collaborative research by undertaking a re-examination of the interpretation of Milano with the objective of elucidating the hitherto obscure geometric structures of frequencies. They obtained the notion of extended frequency, which is a class of mappings on surfaces valued in the Clifford algebra. By emphasising the concept of 'tangential', they showed that the class of generalised frequencies coincides with the class of tangential extended frequencies. They then reached a novel geometric interpretation, whereby the class of generalised frequencies forms an orthogonal geometry. This follows from that the class of extended frequencies constitutes a Clifford algebra-valued vector bundle, on which the orthogonal group acts isometrically.

Biconservative hypersurface and differential equation

CS 13:10

Aykut KAYHAN

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Biconservative hypersurfaces are formulated by the fact that the shape operator acting gradient of the mean curvature is proportional to the gradient of the mean curvature. It is obvious that every minimal or CMC (constant mean curvature) hypersurfaces are biconservative. In this work, we study biconservative hypersurfaces with non-diagonalizable shape operators in Minkowski-4 space and Minkowski 5-space. We started the calculation by assuming that such hypersurfaces are not CMC. Using Gauss Equation and Codazzi equation, by careful analysis, we obtained a relation between the mean curvature and its derivative. Making use of the Local Frobenius Theorem, we constructed a coordinate system. After this process we realized that the mean curvature of such hypersurfaces satisfy the Riccati differential equation. In order to solve the Riccati differential equation you need to know a particular solution. In this work, interestingly, we found that solving the equation in Minkowski 5- space is less complicated than solving in Minkowski 4-space

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Day 2

September 7th, 9:00 AM – 13:35 PM

Curvature invariants of submanifolds and stability of space forms

PS 9:00

Gabriel Eduard VÎLCU

Politehnica University, and Gheorghe Mihoc – Caius Iacob Institute of Mathematical Statistics and Applied Mathematics of the Romanian Academy, Bucharest, Romania

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We present some inequalities involving basic intrinsic and extrinsic invariants of submanifolds in space forms. We provide examples showing that these inequalities are the best possible and obtain some classification results for ideal submanifolds (in the sense of B.-Y. Chen). Finally, we discuss the stability of locally conformal cosymplectic manifolds and generalized S-space forms.

Paracomplex structures and nilpotent structures

PS 10:00

Naoya ANDO

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The conformal Gauss maps of time-like minimal surfaces in the Lorentz-Minkowski 3-space give sections of the time-like twistor spaces associated with the pull-back bundles with fully light-like covariant derivatives. The covariant derivative of the paracomplex structure of the pull-back bundle corresponding to such a section is locally represented by the tensor product of a nowhere zero 1-form and a nilpotent structure. In this talk, we consider paracomplex structures and nilpotent structures of an oriented neutral vector bundle of rank 4n with a metric connection. We have characterizations, examples and so forth of paracomplex structures with the aforementioned property and nilpotent structures with the Walker condition. In addition, we have a characterization of a neutral hyperKähler structure of the vector bundle in terms of a nilpotent structure.

Codimension 2 submanifolds of paracosymplectic manifolds with normal Reeb vector field

PS 11:20

Cornelia-Livia BEJAN

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This study is devoted to a submanifold M of codimension 2 of an almost paracontact metric manifold $\overline{M}$, for which the Reeb vector field of the ambient manifold is normal. Some sufficient conditions for the existence of M are given. When $\overline{M}$ is paracosymplectic, then some necessary and sufficient conditions are established for M to fall in one of the following classes of almost paracontact metric manifolds according to the classification given by S. Zamkovoy and G. Nakova: normal, paracontact metric, para-Sasakian, K-paracontact, quasi-para-Sasakian, respectively. When in addition, M is para-Sasakian and $\overline{M}$ is paracosymplectic, some characterization results are obtained for M to be totally umbilical, as well as a nonexistence result for M to be totally geodesic is provided. The case when $\overline{M}$ is of a constant sectional curvature is analysed and an example is constructed. This is a joint work with professor Galia Nakova (Bulgaria) and the main part of this paper was published in Filomat, 2023.

Comparisons of extended divergence on a foliation by deformed probability simplexes with other divergences

CS 12:20

Keiko UOHASHI

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In the field of nonextensive statistics, q-normal distributions and the generalization, q-exponential families, play an important role. In this study, we deal with deformed probability simplexes corresponding to sets of escort distributions for q-parameters with a certain correspondence to α-parameters of information geometry. So, we describe the dualistic structures and the divergences generated by affine immersions on the deformed probability simplexes corresponding to sets of escort distributions. It also includes topics about Hessian manifolds and their level surfaces. Finally, we propose a new decomposition of an extended divergence on the foliation. Comparisons with previously proposed divergence decomposition theorems, as well as comparisons between extended divergence and the duo Bregman divergence used in machine learning, are also discussed.

Biharmonic homogeneous polynomial maps between spheres

CS 12:45

Rareş-Mircea AMBROSIE

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Alexandru Ioan Cuza University, Iaşi, Romania

In this paper, we first prove a characterization formula for biharmonic maps in Euclidean spheres. Then we focus on maps between spheres whose components are homogeneous polynomials of the same degree, leading to a more specific form for their bitension field. Applying this formula to quadratic maps between spheres, we prove that such a map is non-harmonic biharmonic if and only if it has constant energy density (m + 1)/2. We end by presenting a rigidity result for non-harmonic biharmonic quadratic forms and some classification results.

Semi-invariant submanifolds of (α, p)-golden metric manifolds

CS 13:10

Cristina Elena HREȚCANU

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Ştefan cel Mare University, Suceava, Romania

The main objective of this paper is to study some properties of semi-invariant submanifolds of (α, p)-golden metric manifolds. Some properties of these types of submanifolds, regarding the integrability of the distributions and mixed-geodesic property are presented.

Day 3

September 9th, 9:00 AM – 13:35 PM

Constructing pre-contrast functions that induce specified Bartlett tensor fields

PS 9:00

Hiroshi MATSUZOE

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In information geometry and the geometry of statistical manifolds, an asymmetric squared-distance function, known as a divergence or contrast function, plays a crucial role. The asymmetry of the contrast function induces a dualistic structure of affine connections.
A pre-contrast function generalizes the contrast function and acts as a distance-like function that depends on direction. Pre-contrast functions are particularly useful in quantum information geometry and the statistical theory of nonconservative estimation.
The Bartlett tensor field is a tensor field that has information similar to curvature. In this talk, we consider constructing the pre-contrast function based on the given Bartlett tensor field.

First integrals and symmetries in Finsler geometry

PS 10:00

Ioan BUCATARU

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I discuss a Finslerian version of Noether's theorem, providing the equivalence between $1-$homogeneous first integrals and $0-$homogeneous Cartan symmetries.

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c-Biharmonic maps: a higher order conformally invariant generalization of harmonic maps

PS 11:20

Cezar ONICIUC

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Alexandru Ioan Cuza University, Iaşi, Romania

Biharmonic maps between Riemannian manifolds are critical points of the bienergy functional E_2, and represent a natural generalization of harmonic maps. These maps are characterized by a fourth-order non-linear elliptic equation and exhibit intriguing properties, especially in the case of biharmonic immersions in spheres. However, a limitation of biharmonic maps is their lack of invariance under conformal changes of the domain metric (M^m, g) for any dimension m. In contrast, harmonic maps are conformally invariant when $m = 2$. This suggests the need for a new generalization of harmonic maps that preserves conformal invariance for some m. In this talk, we introduce the concept of c-biharmonic maps. By modifying the bienergy functional with additional lower-order terms, we define a new functional E_2^c, called the c-biharmonic functional, whose critical points are known as c-biharmonic maps. The c -biharmonic equation is also a fourth-order non-linear elliptic equation, but c-biharmonic maps are conformally invariant when $m = 4$. The lower-order modification of the bienergy functional does not change the analytic properties of c-biharmonic maps comparing with the biharmonic ones. On the other hand, from geometric point of view, and especially for isometric immersions, the behavior of c -biharmonic immersions is very different from that of biharmonic ones. This is a joint work with Volker Branding and Simona Nistor.

Finsler metrizabilities and geodesic invariance

CS 12:20

Oana CONSTANTINESCU

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Alexandru Ioan Cuza University, Iaşi, Romania

We demonstrate that various metrizability problems for Finsler sprays can be reformulated in terms of the geodesic invariance of two tensors, namely the metric and angular tensors. We show that a spray S is the geodesic spray of some Finsler metric if and only if its metric tensor is geodesically invariant. Moreover, we establish that gyroscopic sprays constitute the largest class of sprays characterized by a geodesic-invariant angular metric. Scalar functions associated with these geodesically invariant tensors will also be invariant, thereby providing first integrals for the given spray. This talk is based on a joint work with Ioan Bucătaru.

Hardy type inequalitites and the impact of the curvature of the boundary

CS 12:45

Ioan Radu PETER

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We present some Hardy type inequalities on domaines in Minkowski spaces with focus on the impact of the curvature of the domain in the inequalities. Also some applications to existence and unicity problems in PDE are provided.

Gap results for biharmonic submanifolds in Euclidean spheres

CS 13:10

Ştefan ANDRONIC

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The first part of the presentation will cover some general properties of biharmonic submanifolds and, in particular, a gap result for the value of the mean curvature in the case of biharmonic submanifolds with parallel mean curvature vector field (PMC) in Euclidean spheres. Further, we will determine a larger gap of the mean curvature for a class of PMC proper biharmonic submanifolds in spheres. When the bounds of the new gap are reached, we obtain splitting results of the submanifold.
