

HOMework

1 If $A = \begin{pmatrix} 4 & 9 \\ -1 & -2 \end{pmatrix}$, find $A^n, n \geq 2$.

Hint: Write $A = I_2 + B$, with $B^2 = O_2$. Use the binomial formula.

2 Find $D = \begin{vmatrix} x & a & b & c \\ a & x & b & c \\ a & b & x & c \\ a & b & c & x \end{vmatrix}$, where $a, b, c, x \in \mathbb{C}$.

3 Find $D = \begin{vmatrix} a & b & -a & -b \\ -b & a & b & -a \\ c & d & c & d \\ -d & c & -d & -c \end{vmatrix}$, where $a, b, c, d \in \mathbb{C}$.

Hint: Use Laplace rule (expand over the first two rows).

4 Find $a \in \mathbb{R}$ so that the matrix $A = \begin{pmatrix} a+1 & 3 & 1 & 2 \\ -1 & 1 & -1 & 1 \\ a-2 & -2 & 2 & -2 \end{pmatrix}$ has rank 2.

5 Discuss the rank of the matrix $A = \begin{pmatrix} 1+a & a & a \\ a & 1+a & a \\ a & a & 1+a \end{pmatrix}$ with respect to the values of $a \in \mathbb{R}$.

6 Let $A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ and $B = A - I_4$.

1. Prove that $(I_4 - A)(I_4 + A + A^2 + A^3) = I_4$.

2. Prove that B is invertible and find B^{-1} .