1. (a) Prove that $B = \{v_1 = (1,2,3), v_2 = (2,3,1), v_3 = (3,1,2)\}$ is a basis in $\mathbb{R}^3$ and find the coordinates of $v = (2,0,4)$ with respect to this basis.

   (b) Is $B' = \{w_1 = v_1 + v_2, w_2 = v_2 + v_3, w_3 = v_3 - v_1\}$ a basis in $\mathbb{R}^3$? What about $B'' = \{w_1 = v_1 + v_2, w_2 = v_2 + v_3, w_3 = v_3 + v_1, w_4 = v_1 - v_2\}$?

   (c) Find a basis in $\mathbb{R}^3$ such that v has coordinates $1, 0, 1$ with respect to this basis.

Grading: 5p+3p+1p+**1p**

2. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$, $T(x_1, x_2, x_3) = (2x_1 + 3x_2, x_1 - x_2 + x_3, -x_1 + 3x_2 + x_3)$.

   (a) Find the matrix of T with respect to the canonical basis in $\mathbb{R}^3$.

   (b) Find the matrix of T with respect to the basis $B' = \{v_1 = (1,1,1), v_2 = (1,1,0), v_3 = (1,1,1)\}$ in $\mathbb{R}^3$.

   (c) Prove that T is injective and surjective.

   (d) Are there $x = (x_1, x_2, x_3)$, $y = (y_1, y_2, y_3)$ in $\mathbb{R}^3$, $x \neq y$, such that $Tx = Ty = (8,9,10)$?

Grading: 2p+2p+4p+1p+**1p**

3. (a) Give an example of a linear subspace of $\mathbb{R}^4$ of dimension 2.

   (b) Let $S = \left\{ \begin{pmatrix} a & b \\ 2b & a \end{pmatrix} ; a, b \in \mathbb{R} \right\}$. Prove that S, equipped with the standard matrix addition and scalar matrix multiplication, is a linear subspace of $M_2(\mathbb{R})$ and find its dimension, together with a basis in S. Is S isomorphic to $\mathbb{R}^3$?

   (c) Find a linear subspace W of $M_2(\mathbb{R})$, $W \neq \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \right\}$, such that $S \cap W = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \right\}$

   (d) Is there any linear subspace of S with dimension 10?

   (e) Is it possible to find a system of 10 linearly independent vectors in S?

Grading: 1p+5p+1p+1p+1p+**1p**

Each problem is graded from 1 to 10. The final grade is the arithmetic mean of the grades received for each problem. Allotted time: 100 minutes.