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PERIODIC OSCILLATIONS AND BIFURCATION ANALYSIS FOR A COHEN-GROSSBERG NEURAL NETWORK MODEL WITH IMPULSIVE PERTURBATIONS

BY

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Abstract. This paper investigates the behavior of a Cohen-Grossberg neural network composed of two neurons which are subject to periodic impulsive perturbations. By employing Mawhin's continuation theorem, we determine sufficient conditions for the existence of semi-trivial periodic solutions. The asymptotic stability of these solutions is then investigated using the Floquet theory of impulsive differential equations. Finally, we discuss the bifurcation of nontrivial periodic solutions with the help of a projection method.

Key words: neural network, Cohen-Grossberg model, periodic impulsive perturbations, semi-trivial periodic solution, stability, bifurcation.

1. Introduction

Cohen and Grossberg proposed and investigated (Cohen & Grossberg, 1983) a model of a self-organizing neural network which describes the short-term storage of visual and language patterns, in the following form

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$$x_i' = a_i(x_i)\left(b_i(x_i) - \sum_{j=1}^{n} t_{ij} s_j(x_j) + J_i\right), \quad i = 1,2,\ldots,n. \tag{1}$$

Here, $n \geq 2$ is the total number of neurons inside the network, x_i is the state variable associated to the i-th neuron, a_i is an amplification (or self-inhibition, depending on whether it has positive or negative sign) function, b_i is an appropriately behaved function describing the rate at which the i-th neuron self-regulates its potential when isolated from the network and J_i denotes the constant input from outside of the network, also called external bias. The n x n matrix $T = [t_{ij}]$, supposed to be symmetric and to have positive entries (Cohen & Grossberg, 1983), indicates how the neurons are interconnected in the network, namely whether the output from the j-th neuron excites or inhibits the i-th neuron, while also characterizing the strength of the connection between neurons. The activation function s_j describes how the j-th neuron reacts to the input. It has also been noted (Cohen & Grossberg, 1983) that the system (1) formally includes several usual population dynamics models such as n-species Lotka-Volterra and Gilpin-Ayala models of competitive interaction.

In the real world, it is often the case that evolutionary processes are followed by abrupt changes, caused by switching phenomena, frequency modulations, or other unexpected perturbations. Such occurrences can be meaningfully represented by using impulsive dynamical systems, characterized by the coexistence of continuous and discrete dynamics. In this regard, if the impulsive perturbations occur whenever the trajectories of the solutions reach a prescribed subset of the state space, the corresponding impulsive dynamical system is called state-dependent, while if the impulsive perturbations occur at prescribed time instances, independent of the system state, then the corresponding impulsive dynamical system is called time-dependent.

In the past few years, researchers have gradually come to the conclusion that the influence of impulsive perturbations should not be ignored when describing the behavior of Cohen-Grossberg neural networks, and obtained results describing the asymptotic behavior, global exponential stability and existence of periodic solutions (Gopalsamy, 2004; Zhang & Chen, 2008; Chen & Ruan, 2005; Yang & Xu, 2006; Song & Zhang, 2008; Chen & Ruan, 2007; Yang & Cao, 2007; Zhang & Luo, 2013 and the references therein). It was also noted that in concrete problems periodic solutions correspond to emerging storage or memory patterns. However, comparatively few efforts have been devoted to the analysis of bifurcation phenomena in impulsive neural networks. In this regard, in order to obtain a deep understanding of the dynamics of neural networks, many authors have focused on simple systems, namely on bineuronal networks (Huang & Wu, 2001a; Huang & Wu, 2001b; Guo *et al*., 2008; Hsu *et* *al*., 2010; Zhou *et al*., 2009; Du *et al*., 2013). Despite of their low neuron count, impulsively perturbed bineuronal networks display in many situations the same behavior as large networks, being suitable as working prototypes in order to improve our understanding pertaining to the influence of impulses.

In this paper, we consider the following impulsive differential system describing a bineuronal network:

$$\begin{cases} x_1' = -a_1(x_1(t))\left[b_1(x_1(t)) - h_{11}f_1\left(x_1(t)\right) - h_{12}f_2\left(x_2(t)\right) + I_1\right], \\ x_2' = -a_2(x_2(t))\left[b_2(x_2(t)) - h_{21}f_1\left(x_1(t)\right) - h_{22}f_2\left(x_2(t)\right) + I_2\right], \\ \qquad t \neq (n+\widetilde{l}-1)T, t \neq nT, \\ \qquad \Delta x_1(t) = 0, \\ \qquad \Delta x_2(t) = p_2 x_2(t), \\ \qquad t = (n+\widetilde{l}-1)T, \\ \qquad \Delta x_1(t) = p_1 x_1(t), \\ \qquad \Delta x_2(t) = 0, \\ \qquad t = nT. \end{cases} \tag{2}$$

Here, x_i is the state variable associated to the *i*-th neuron, $i = 1,2$, $n \in \mathbf{N}^*$, $T > 0$ represents the common periodicity of both impulsive perturbations and $0 < \widetilde{l} < 1$ is used to characterize the intervals of time between them, of length $\widetilde{l}\,T$ and $(1-\widetilde{l}\,)T$, respectively. Also, $\Delta x_i(t) = x_i(t+) - x_i(t)$, $i = 1,2$, represents the instantaneous jump in the state of the *i*-th neuron as a result of the impulsive perturbations at time t, $t = (n+\widetilde{l}-1)T$ or $t = nT$. Consequently, it is assumed that each neuron is activated in a periodic fashion, with same periodicity but at different moments.

The coefficients p_i, $i = 1,2$, are real constants which quantify the relative magnitude of the impulses, such that $p_i > -1$, $i = 1,2$. The functions $a_i : \mathbf{R} \to \mathbf{R}$, $i = 1,2$, and, respectively, $b_i : \mathbf{R} \to \mathbf{R}$, $i = 1,2$, are $C^1(\mathbf{R})$ functions which satisfy the following growth assumptions.

(H1) There exist $m_i, M_i \geq 0$, $i = 1,2$, such that

$$m_i u \leq a_i(u) \leq M_i u, \quad \forall u \in \mathbf{R}, i = 1,2.$$

(H2) There exist $c_i, d_i \geq 0$, $i = 1,2$, such that

$$c_i u \le b_i(u) \le d_i u, \quad \forall u \in \mathbf{R}, i = 1,2.$$

The general activation functions $f_i : \mathbf{R} \to \mathbf{R}$ are also $C^1(\mathbf{R})$ functions, assumed to satisfy the following sublinear growth condition

(H3) There exist $k_i, r_i \ge 0$, $i = 1,2$, such that

$$|f_i(x)| \le k_i |x| + r_i, \quad \forall x \in \mathbf{R}, i = 1,2.$$

The remaining part of this paper is organized as follows. In Section 2, we state some notations and basic definitions which are necessary to state the continuation theorem, while also establishing sufficient conditions for the existence of semi-trivial periodic solutions. The stability of these semi-trivial periodic solutions is investigated in Section 3. It is then shown in Section 4 that once a threshold condition is reached, the semi-trivial solution loses its stability and a nontrivial periodic solution appears via a bifurcation phenomenon. Finally, a brief discussion of our main findings is given in Section 5.

2. Preliminaries

2.1. The Continuation Theorem

In this subsection, we introduce certain notions relating to Gaines and Mawhin's coincidence degree theory.

Definition 1. *Let* $(X, \|\cdot\|)_X$ *and* $(Z, \|\cdot\|)_Z$ *be real Banach spaces and let* $L : D(L) \subset X \to Z$ *be a linear operator. The operator* L *is called a Fredholm operator of index zero if* $\dim(\operatorname{Ker} L) = \operatorname{codim}(\operatorname{Im} L) < \infty$ *and* $\operatorname{Im} L$ *is closed in* Z.

If L is a Fredholm operator of index zero and there exist continuous projectors $P : X \to X$ and $Q : Z \to Z$ such that

$$\operatorname{Im} P = \operatorname{Ker} L, \quad \operatorname{Ker} Q = \operatorname{Im} L = \operatorname{Im}(I - Q), \tag{3}$$

I being the identity of Z, then $L|_{\operatorname{Dom} L \cap \operatorname{Ker} P} : (I - P)X \to \operatorname{Im} L$ is invertible. Let us denote its inverse by K_P. Since $\operatorname{Im} Q$ is isomorphic to $\operatorname{Ker} L$, there also exists an isomorphism $J : \operatorname{Im} Q \to \operatorname{Ker} L$.

Definition 2. *If* $N : X \to Z$ *is a continuous operator,* Ω *is an open bounded set of* X *and* L *is a Fredholm operator of index zero such that* (*3*) *is* *satisfied, the operator* N *is called L-compact on* $\overline{\Omega}$ *if* $QN(\overline{\Omega})$ *is bounded and* $K_P(I-Q)N:\overline{\Omega}\to X$ *is compact.*

We are now ready to state Mawhin's continuation theorem.

Lemma 1. (Gaines & Mawhin, 1977). *Let* $\Omega \subset X$ *be an open bounded set, let* L *be a Fredholm operator of index zero and let* N *be L-compact on* $\overline{\Omega}$. *Assume that*

(*i*) *for each* $\lambda \in (0,1)$ *and* $x \in \partial\Omega \cap \mathrm{Dom}\,(L)$, $Lx \neq \lambda Nx$,

(*ii*) *for each* $\partial\Omega \cap \mathrm{Ker}\,L$, $QNx \neq 0$,

(*iii*) $\deg(JQN, \Omega \cap \mathrm{Ker}\,L, 0) \neq 0$.

Then the operatorial equation $Lx = Nx$ *has at least one solution in* $\overline{\Omega} \cap \mathrm{Dom}\,L$.

2.2. The Existence of Semi-Trivial T-Periodic Solutions

In this subsection, we shall consider the following reduced subsystem of (2), corresponding to the situation in which $x_2 = 0$

$$\begin{cases} x_1' = -a_1(x_1(t))\left[b_1(x_1(t)) - h_{11}f_1(x_1(t)) - h_{12}f_2(0) + I_1\right], & t \neq nT, \\ x_1(t+) = (1+p_1)x_1(t), & t = nT. \end{cases} \tag{4}$$

Using the change of variables given by $\ln x_1 = x$, the system (4) is transformed into

$$\begin{cases} x' = -\dfrac{a_1(e^{x(t)})}{e^{x(t)}}\left[b_1(e^{x(t)}) - h_{11}f_1(e^{x(t)}) - h_{12}f_2(0) + I_1\right], & t \neq nT, \\ x(t+) = \ln(1+p_1) + x(t), & t = nT. \end{cases} \tag{5}$$

Having in view that we are searching for *T*-periodic solutions of (2), let us introduce the following functional spaces whose definition embeds *T*-periodicity

$$C_p([0,T],\mathbf{R}) = \{u : [0,T] \to \mathbf{R},\ u \text{ is continuous on } (0,T), \text{continuous from the left in } t = T \text{ and } \lim_{t\downarrow 0} u(t) \text{ is finite}\}$$

$$X = \{x \in C_p([0,T],\mathbf{R}) \mid x(0) = x(T)\}, \quad Z = X \times \mathbf{R},$$

and define

$$\| x \|_X = \sup_{t\in[0,T]} |x(t)|, \quad \|(x,r)\|_Z = \| x \|_X + |r|.$$

It is easy to check that $(X, \|\cdot\|_X)$ and $(Z, \|\cdot\|_Z)$ are both Banach spaces. To set the system (5) into the framework of the continuation theorem, let us also define a linear operator L and a nonlinear operator N by

$$L : \mathrm{Dom}\, L \subset X \to Z, \quad Lx = (\Im_1, x(0+) - x(T)),$$
$$\Im_1(t) = x'(t),$$
$$N : X \to Z, \quad Nx = (\Im_2, \ln(1 + p_1)),$$
$$\Im_2(t) = -\frac{a_1(e^{x(t)})}{e^{x(t)}}\left[b_1(e^{x(t)}) - h_{11} f_1\left(e^{x(t)}\right) - h_{12} f_2(0) + I_1\right]$$

One may explicitely compute $\mathrm{Ker}\, L$ and $\mathrm{Im}\, L$ as being

$$\mathrm{Ker}\, L = \{x; \exists C \in \mathbf{R} \text{ such that } x(t) = C \text{ for all } t \in [0,T]\},$$
$$\mathrm{Im}\, T = \left\{(\Im, a) \in Z \mid \int_0^T \Im(s)ds + a = 0\right\}$$

and find that

$$\dim \mathrm{Ker}\, L = 1 = \mathrm{codim}\, \mathrm{Im}\, L.$$

Then $\mathrm{Im}\, L$ is closed in Z and L is a Fredholm operator of index zero. Let us define the projection operators

$$P{:}X \to X, \quad (Px)(t) = \frac{1}{T}\int_0^{\mathrm{T}} x(s)ds, \quad t \in [0,T],$$
$$Q : Z \to Z, \quad Qz = Q(\Im, a) = (\Re, 0),$$
$$\Re(\mathrm{t}) = \frac{1}{T}\left(\int_0^T \Im(s)ds + a\right), \quad t \in [0,T].$$

It is easy to show that P and Q are continuous projectors satisfying

$$\mathrm{Im}\, P = \mathrm{Ker}\, L, \ \mathrm{Im}\, L = \mathrm{Ker}\, Q = \mathrm{Im}\, (I\text{-}Q).$$

Let us now derive an explicit expression for K_P. To this purpose, let $z = (\Im, a) \in \mathrm{Im}\, L$. Then there is $x \in \mathrm{Ker}\,\mathrm{P}$ such that

$$x'(t) = \Im(t), t \in (0,T], \quad x(0+) - x(T) = a,$$

which implies that

$$x(t) = x(0+) + \int_0^T \Im(s)ds = x(0) + a + \int_0^T \Im(s)ds, \quad t \in (0,T].$$

Consequently,

$$x(t) = x(0) + \int_0^T \Im(s)ds - a\left[-\frac{t}{T}\right], \quad t \in (0,T]. \tag{6}$$

in which $\left[-\frac{t}{T}\right]$ denotes the integer part of $\left[-\frac{t}{T}\right]$. Since $x \in \operatorname{Ker} P$, it is seen that $\frac{1}{T}\int_0^T x(t)dt = 0$, which yields using (6) that

$$\int_0^T \int_0^t \Im(s)dsdt + T\big(x(0) + a\big) = 0.$$

Consequently,

$$x(t) = \int_0^t \Im(s)ds - \frac{1}{T}\int_0^T \int_0^t \Im(s)dsdt - a - a\left[-\frac{t}{T}\right], \quad t \in [0,T],$$

and the explicit expression of K_P is given by

$$\big(K_P z\big)(t) = \int_0^t \Im(s)ds - \frac{1}{T}\int_0^T \int_0^t \Im(s)dsdt - a - a\left[-\frac{t}{T}\right]. \tag{7}$$

One then sees using (7) that

$$QNx(t) = \left(\frac{1}{T}\left(\int_0^T \Im_2(t)dt + \ln(1+p_1)\right), 0\right),$$

$$K_P(I-Q)Nx = \int_0^t \Im_2(s)ds - \left[-\frac{t}{T}\right]\ln(1+p_1) - \frac{1}{T}\int_0^T \int_0^t \Im_2(s)dsdt$$
$$- \ln(1+p_1) - \left(\frac{t}{T} - \frac{1}{2}\right)\left(\int_0^T \Im_2(t)dt + \ln(1+p_1)\right).$$

Clearly, QN and $K_P(I\text{-}Q)N$ are continuous operators. It is easy to show that $\overline{K_P(I\text{-}Q)N(\overline{\Omega})}$ is compact for any open bounded subset $\Omega \subset X$.

Moreover, $QN(\overline{\Omega})$ is bounded. Thus, N is L-compact on $\overline{\Omega}$ for any open bounded set $\Omega \subset X$.

We are now in position to search for an appropriate open and bounded subset Ω for the application of the continuation theorem. The operatorial equation $Lx = \lambda Nx$, $\lambda \in (0,1)$, reduces to

$$\begin{cases} x' = -\dfrac{a_1(e^{x(t)})}{e^{x(t)}}\left[b_1(e^{x(t)}) - h_{11}f_1\left(e^{x(t)}\right) - h_{12}f_2(0) + I_1\right], \quad t \in (0,T], \\ \Delta x(t) = \lambda \ln(1+p_1)x(t), \quad t = 0. \end{cases} \tag{8}$$

Suppose that $x \in X$ is a solution of (8) for certain $\lambda \in (0,1)$. Integrating (8) over the interval $[0,T]$, one obtains

$$\begin{aligned} &\int_0^T \frac{a_1(e^{x(t)})}{e^{x(t)}}\left[b_1(e^{x(t)}) - h_{11}f_1\left(e^{x(t)}\right)\right]dt \\ &\quad = \ln(1+p_1) + \left(h_{12}f_2(0) - I_1\right)\int_0^T \frac{a_1(e^{x(t)})}{e^{x(t)}}\,dt. \end{aligned} \tag{9}$$

From **(H2)** and **(H3)**, one sees that

$$b_1(e^{x(t)}) - h_{11}f_1\left(e^{x(t)}\right) \le \left(c_1 - k_1|h_{11}|\right)e^{x(t)} - r_1|h_{11}|. \tag{10}$$

Assume that

$$\begin{cases} c_1 - k_1|h_{11}| > 0, \\ \ln(1+p_1) - M_1T\left|-r_1|h_{11}| + h_{12}f_2(0) - I_1\right| > 0. \end{cases} \tag{11}$$

It then follows from **(H1)**, **(H3)**, (8) and (9) that

$$\begin{aligned} \int_0^T |x'(t)|dt &\le \lambda\int_0^T \frac{a_1(e^{x(t)})}{e^{x(t)}}\left|b_1(e^{x(t)}) - h_{11}f_1\left(e^{x(t)}\right) + r_1|h_{11}|\right|dt \\ &\quad + \lambda\int_0^T \frac{a_1(e^{x(t)})}{e^{x(t)}}\left|I_1 - h_{12}f_2(0) - r_1|h_{11}|\right|dt \\ &\le \lambda\int_0^T \frac{a_1(e^{x(t)})}{e^{x(t)}}\left|b_1(e^{x(t)}) - h_{11}f_1\left(e^{x(t)}\right) + r_1|h_{11}|\right|dt \\ &\quad + \lambda M_1T\left|I_1 - h_{12}f_2(0) - r_1|h_{11}|\right| \\ &\le \ln(1+p_1) + 2\left|I_1 - h_{12}f_2(0)b_1(e^{x(t)}) - r_1|h_{11}|\right|. \end{aligned} \tag{12}$$

Since $x \in X$, there exist $\xi, \eta \in [0, T]$ such that

$$x(\xi +) = \inf_{t \in [0,T]} x(t), \quad x(\eta +) = \sup_{t \in [0,T]} x(t), \tag{13}$$

in the case in which one of them equals T the respective limit being understood to be replaced by $x(T)$. Then

$$\begin{aligned} x(t) &\le x(\xi +) + \int_0^T |x'(s)| ds \\ &\le x(\xi +) + \ln(1 + p_1) + 2\left|I_1 - h_{12} f_2(0) - r_1 |h_{11}|\right|. \end{aligned} \tag{14}$$

Using (9) and (10), it follows that

$$\begin{aligned} &\int_0^T \frac{a_1(e^{x(t)})}{e^{x(t)}} \left[(c_1 - k_1 |h_{11}|) e^{x(t)} \right] dt \\ &\quad \le \ln(1 + p_1) + M_1 T \left| r_1 |h_{11}| + h_{12} f_2(0) - I_1 \right|, \end{aligned}$$

and consequently

$$\begin{aligned} &m_1 e^{x(\xi +)} (c_1 - k_1 |h_{11}|) T \\ &\quad \le \ln(1 + p_1) + M_1 T \left| r_1 |h_{11}| + h_{12} f_2(0) - I_1 \right|, \end{aligned}$$

so

$$x(\xi +) \le \ln \left(\frac{\ln(1 + p_1) + M_1 T \left| r_1 |h_{11}| + h_{12} f_2(0) - I_1 \right|}{m_1 (c_1 - k_1 |h_{11}|) T} \right). \tag{15}$$

Then from (14) and (15), one sees that

$$\begin{aligned} x(t) &\le \ln \left(\frac{\ln(1 + p_1) + M_1 T \left| r_1 |h_{11}| + h_{12} f_2(0) - I_1 \right|}{m_1 (c_1 - k_1 |h_{11}|) T} \right) \\ &\quad + \ln(1 + p_1) + 2\left|I_1 - h_{12} f_2(0) - r_1 |h_{11}|\right| = \widetilde{H}_1. \end{aligned} \tag{16}$$

By a similar argument,

$$\begin{aligned} x(t) &\ge x(\eta +) - \int_0^T |x'(s)| ds \\ &\ge x(\eta +) - \ln(1 + p_1) + 2\left|I_1 - h_{12} f_2(0) - r_1 |h_{11}|\right| \end{aligned} \tag{17}$$

and

$$b_1(e^{x(t)}) - h_{11} f_1(e^{x(t)}) \le \left(d_1 + k_1 |h_{11}|\right) e^{x(t)} + r_1 |h_{11}|.$$

It is then seen that

$$\int_0^T \frac{a_1(e^{x(t)})}{e^{x(t)}} \left(d_1 + k_1 |h_{11}|\right) e^{x(t)} \ge \ln(1+p_1) - M_1 T \left| - r_1 |h_{11}| + h_{12} f_2(0) - I_1 \right|,$$

which implies

$$M_1 e^{x(\eta+)} \left(d_1 + k_1 |h_{11}|\right) T \ge \ln(1+p_1) - M_1 T \left| - r_1 |h_{11}| + h_{12} f_2(0) - I_1 \right|,$$

and consequently

$$x(\eta+) \ge \ln\left(\frac{\ln(1+p_1) - M_1 T \left| - r_1 |h_{11}| + h_{12} f_2(0) - I_1 \right|}{M_1 \left(d_1 + k_1 |h_{11}|\right) T} \right). \tag{18}$$

Then from (17) and (18), one sees that

$$\begin{aligned} x(t) \ge {} & \ln\left(\frac{\ln(1+p_1) - M_1 T \left| - r_1 |h_{11}| + h_{12} f_2(0) - I_1 \right|}{M_1 \left(d_1 + k_1 |h_{11}|\right) T} \right) \\ & - \ln(1+p_1) - 2 M_1 T \left| I_1 - h_{12} f_2(0) - r_1 |h_{11}| \right| = \widetilde{H}_2. \end{aligned} \tag{19}$$

Clearly, $\widetilde{H}_i$, $i = 1,2$, are independent of λ. Take

$$\widetilde{H} = \max\left(\widetilde{H}_1, \widetilde{H}_2\right) + 1.$$

Then $|x|_T = \sup_{t \in [0,T]} |x(t)| < \widetilde{H}$ whenever $x \in X$ is a solution of (8) for any $\lambda \in (0,1)$.

Define $\Omega = \left\{ x \in X : |x|_T < \widetilde{H} \right\}$. Then, from the above estimations, there are no $\lambda \in (0,1)$ and $x \in \partial\Omega$ such that $Lx = Nx$. Also, if $x \in \partial\Omega \cap \operatorname{Ker} L$, then $x(t) = \alpha$, $t \in [0,T]$, with $|x|_T = \widetilde{H}$. It follows that

$$\begin{aligned} QNx = {} & \left(-\frac{a_1(e^\alpha)}{e^\alpha} \left[b_1(e^\alpha) - h_{11} f_1(e^\alpha) - h_{12} f(0) + I_1 \right] + \frac{1}{T} \ln(1+p_1), 0 \right) \\ & \ne 0, \end{aligned}$$

since otherwise $|x|_T \leq \max(\widetilde{H}_1, \widetilde{H}_2) < \widetilde{H}$.

It is easily seen that $(JQN)^{-1}(0) \cap (\Omega \cap \operatorname{Ker} L) \neq \varnothing$. Consider the homotopy

$$\Theta : (\Omega \cap \operatorname{Ker} L) \times [0,1] \to \Omega \cap \operatorname{Ker} L,$$

$$\Theta(x,\mu) = -\frac{a_1(e^x)}{e^x}\left[x - h_{12} f_2(0) + I_1\right] + \frac{1}{T}\ln(1+p_1)$$
$$-\mu \frac{a_1(e^x)}{e^x}\left[b_1(e^x) - h_{11} f_1(e^x) - x\right].$$

Note that $\Theta(x,1) = JQN$. If $\Theta(x,\mu) = 0$, then we get $|x|_T < \widetilde{H}$. Hence,

$$\Theta(x,\mu) \neq 0 \ \text{ for } (x,\mu) \in (\Omega \cap \operatorname{Ker} L) \times [0,1].$$

It follows from the property of invariance of degree under a homotopy that

$$\deg(JQN, \Omega \cap \operatorname{Ker} L, 0) = -1 \neq 0.$$

Combining the above analysis with the conclusions of Lemma 3, one obtains the following existence result.

Theorem 4. *Suppose that conditions* **(H1)-(H3)** *and* (11) *hold. Then the system* (4) *has a T-periodic solution* $x_1^*(t)$.

Restating the above theorem for the system (2), one establishes the existence of its semi-trivial periodic solutions under the above-mentioned sufficient conditions.

Theorem 5. *Suppose that conditions* **(H1)-(H3)** *and* (11) *hold. Then the system* (2) *has a T-periodic solution* $(x_1^*(t), 0)$.

Note that the second part of condition (11) states that, in order to steer the system (2) to a semi-trivial *T*-periodic solution with nonzero x_1, the impulsive perturbation of x_1 needs to have its relative magnitude p_1 larger than a certain value, which is certainly conceivable.

Remark 6. *Let us consider another subsystem of the system* (2) *corresponding this time to the situation in which* $x_1 = 0$*, namely*

$$\begin{cases} x_2' = -a_2(x_2(t))\left[b_2(x_2(t)) - h_{21} f_1(x_1(t)) - h_{22} f_2(x_2(t)) + I_2\right], \\ \qquad t \neq (n + \widetilde{l} - 1)T, \\ x_2(t+) = (1+p_2)x_2(t), \\ \qquad t = (n + \widetilde{l} - 1)T. \end{cases} \tag{20}$$

By using an argument similar to the one employed above, one finds that if conditions **(H1)-(H3)** *and*

$$\begin{cases} c_2 - k_2|h_{22}| > 0, \\ \ln(1+p_2) - M_2T\left| - r_2|h_{22}| + h_{21}f_1(0) - I_2 \right| > 0. \end{cases} \tag{21}$$

hold, then the system (2) *has a semi-trivial T-periodic solution* $(0, x_2^*(t))$.

Similarly, the second half of (21) states that the impulsive perturbation of x_2 needs to have its relative magnitude p_2 larger enough for a semi-trivial T-periodic solution with nonzero x_2 to occur.

3. The Stability of Semi-Trivial *T*-Periodic Solutions

We now discuss the asymptotic stability of the semi-trivial periodic solution $(x_1^*(t), 0)$, whose existence has been obtained in the previous section, by means of using the method of small amplitude perturbations. To this purpose, let $(x_1(t), x_2(t))$ be a solution of (2) and let

$$x_1(t) = v(t) + x_1^*(t), \quad x_2(t) = u(t),$$

where *u, v* are understood to be small amplitude perturbations. With these notations, the right-hand sides of the first two eqs. in (2) can now be expanded using Taylor series. After neglecting the higher-order terms, the linearized equations, together with their corresponding impulsive perturbations, read as

$$\begin{cases} v'(t) = \theta_1(x_1^*(t))v + \theta_2(x_1^*(t))u, \\ u'(t) = \theta_3(x_1^*(t))u, \\ t \neq (n+\tilde{l}-1)T, t \neq nT, \\ \Delta v(t) = 0, \\ \Delta u(t) = p_2 u(t), \\ t = (n+\tilde{l}-1)T, \\ \Delta v(t) = p_1 v(t), \\ \Delta u(t) = 0, \\ t = nT. \end{cases} \tag{22}$$

The functional coefficients θ_1, θ_2, θ_3 which appear in the above equation are given by

$$\begin{cases} \theta_1(x_1^*) = -a_1(x_1^*)\big(b_1'(x_1^*) - h_{11} f_1'(x_1^*)\big) \\ \qquad\qquad - a_1'(x_1^*)\big(b_1(x_1^*) - h_{11} f_1(x_1^*) - h_{12} f_2(0) + I_1\big), \\ \theta_2(x_1^*) = a_1(x_1^*) h_{12} f_2'(0), \\ \theta_3(x_1^*) = -a_2'(0)\big(b_2(0) - h_{21} f_1(x_1^*) - h_{22} f_2(0) + I_2\big). \end{cases} \tag{23}$$

Let $M(t)$ be a fundamental matrix of the subsystem constructed with the first two eqs. in (22). Then $M(t)$ satisfies

$$\begin{cases} M'(t) = \begin{pmatrix} \theta_1(x_1^*(t)) & \theta_2(x_1^*(t)) \\ 0 & \theta_3(x_1^*(t)) \end{pmatrix} M(t), \\ \qquad\qquad t \neq (n + \widetilde{l} - 1)T,\ t \neq nT, \\ M(t+) = \begin{pmatrix} 1 + p_1 & 0 \\ 0 & 1 \end{pmatrix} M(t), \\ \qquad\qquad t = nT, \\ M(t+) = \begin{pmatrix} 1 & 0 \\ 0 & 1 + p_2 \end{pmatrix} M(t), \\ \qquad\qquad t = (n + \widetilde{l} - 1)T. \end{cases}$$

Consequently, the following upper triangular matrix M^* is a monodromy matrix of (22),

$$M^* = \begin{pmatrix} d_{11} & d_{12} \\ 0 & d_{22} \end{pmatrix},$$

in which

$$\begin{cases} d_{11} = (1 + p_1) e^{\int_0^T \theta_1(x_1^*(s)) ds}, \\ d_{12} = (1 + p_1)\left(p_2 \int_{\widetilde{l}T}^{T} \theta_2\big(x_1^*(s)\big) e^{\int_0^s \theta_3(x_1^*(\xi)) d\xi + \int_s^T \theta_1(x_1^*(\xi)) d\xi} ds \right) \\ \qquad + (1 + p_1)\left(\int_0^{\widetilde{l}T} \theta_2\big(x_1^*(s)\big) e^{\int_0^s \theta_3(x_1^*(\xi)) d\xi + \int_s^T \theta_1(x_1^*(\xi)) d\xi} ds \right), \\ d_{22} = (1 + p_2) e^{\int_0^T \theta_3(x_1^*(s)) ds}. \end{cases}$$

Since M^* is upper triangular, its eigenvalues are

$$\lambda_1 = d_{11} = (1+p_1)e^{\int_0^T \theta_1\left(x_1^*(s)\right)ds} > 0,$$
$$\lambda_2 = d_{22} = (1+p_2)e^{\int_0^T \theta_3\left(x_1^*(s)\right)ds} > 0.$$

As seen from the Floquet theory of impulsive differential equations (Bainov & Simeonov, 1993), the semi-trivial periodic solution $(x_1^*(t),0)$ is then asymptotically stable if and only if $|\lambda_i| < 1$, $i = 1,2$, that is,

$$(1+p_1)e^{\int_0^T \theta_1\left(x_1^*(s)\right)ds} < 1 \quad \text{and} \quad (1+p_2)e^{\int_0^T \theta_3\left(x_1^*(s)\right)ds} < 1. \tag{24}$$

By the above argument, one obtains the following stability result.

Theorem 7. *Assume that conditions* **(H1)-(H3)**, (11) *and* (24) *hold. Then the system* (2) *has a semi-trivial T-periodic solution* $(x_1^*(t),0)$, *which is asymptotically stable.*

It can also be noted that if the opposite of any of the inequalities in (24) holds, then the semi-trivial *T*-periodic solution $(x_1^*(t),0)$ becomes unstable.

Corollary 8. *Assume that conditions* **(H1)-(H3)**, (11) *and*

$$(1+p_1)e^{\int_0^T \theta_1\left(x_1^*(s)\right)ds} > 1 \quad \text{or} \quad (1+p_2)e^{\int_0^T \theta_3\left(x_1^*(s)\right)ds} > 1. \tag{25}$$

hold. Then the system (2) *has a semi-trivial T-periodic solution* $(x_1^*(t),0)$, *which is unstable.*

Remark 9. *As far as the effect of the impulsive perturbations is concerned, it is useful to note that a large* p_i, $i = 1,2$, *may always destabilize the the semi-trivial T-periodic solution* $(x_1^*(t),0)$ *by bringing* $1+p_1$ (*or* $1+p_2$) *above* $e^{-\int_0^T \theta_1\left(x_1^*(s)\right)ds}$ (*or* $e^{-\int_0^T \theta_3\left(x_1^*(s)\right)ds}$). *This is natural, since impulsive perturbations of large relative magnitude have the potential to destabilize periodic dynamics. In this regard, conditions* (11) *and* (24) *employed in the statement of Theorem 7 impose opposite bounds on the relative magnitude of the perturbation* p_1. *Actually, depending on the particulars of the system* (2), *the existence condition* (11), *which requires a lower bound, and the stability condition* (24), *which requires an upper bound, may be mutually exclusive.*

A similar argument can be pursued in regard to the stability of the other semi-trivial periodic solution $(0, x_2^*)$, using this time the corresponding linearization of (2) near $(0, x_2^*)$ and condition (21) instead of (11).

In the case in which the positive eigenvalues of the monodromy matrix $(1+p_1)e^{\int_0^T \theta_1(x_1^*(s))ds}$ and $(1+p_2)e^{\int_0^T \theta_3(x_1^*(s))ds}$ are close to 1, the impulsive perturbations have a significant effect on the behavior of the neural network. In the following, we investigate the following cases.

Case 1 $(1+p_1)e^{\int_0^T \theta_1(x_1^*(s))ds} \neq 1,\ (1+p_2)e^{\int_0^T \theta_3(x_1^*(s))ds} = 1;$

Case 2 $(1+p_1)e^{\int_0^T \theta_1(x_1^*(s))ds} \neq 1,\ (1+p_2)e^{\int_0^T \theta_3(x_1^*(s))ds} = 1.$

4. The Bifurcation of a Nontrivial Periodic Solution

Until now, we have discussed the existence and stability of the semi-trivial T-periodic solution $(x_1^*(t), 0)$. We are now interested in the bifurcation of nontrivial periodic solutions near $(x_1^*(t), 0)$.

First, we shall denote by $\Phi(t; U_0)$ the solution of the unperturbed system corresponding to the system (2) with the initial data $U_0 = (u_0^1, u_0^2)$. Let us also denote $\Phi = (\Phi_1, \Phi_2)$. We define two maps $\Theta_1, \Theta_2 : \mathbf{R}^2 \to \mathbf{R}^2$ to describe the effects of impulsive perturbations by

$$\Theta_1(x_1, x_2) = (x_1, (1+p_2)x_2), \quad \Theta_2(x_1, x_2) = ((1+p_1)x_1, x_2)$$

and the map $F : \mathbf{R}^2 \to \mathbf{R}^2$ to incorporate the right-hand sides of the unperturbed system by

$$F(x_1, x_2) = \big(-a_1(x_1(t))\big[b_1(x_1(t)) - h_{11} f_1(x_1(t)) - h_{12} f_2(x_2(t)) + I_1\big],$$
$$-a_2(x_2(t))\big[b_2(x_2(t)) - h_{21} f_1(x_1(t)) - h_{22} f_2(x_2(t)) + I_2\big]\big).$$

Next, we shall reduce the problem of finding a periodic solution of (2) to a certain fixed point problem. See also Georgescu *et al.* (2008) or Zhang *et al.* (2008) for related arguments. To this purpose, define $\Psi : [0,\infty) \times \mathbf{R}^2 \to \mathbf{R}^2$ by

$$\Psi(t, U_0) = \Theta_2\Big(\Phi\big((1-\tilde{l})T; \Theta_1\big(\Phi(\tilde{l}T, U_0)\big)\big)\Big),$$

and denote

$$\Psi(t,U_0) = \left(\Psi_1(t,U_0), \Psi_2(t,U_0)\right).$$

Then U is a T-periodic solution of system (2) if and only if its initial data $U(0) = U_0$ is a fixed point for the operator Ψ. As previously seen,

$$D_X \Psi(T, X_0) = \begin{pmatrix} d_{11} & d_{12} \\ 0 & d_{22} \end{pmatrix},$$

in which d_{11}, d_{12} and d_{22} are given by (24).

To find a nontrivial periodic solution of period τ with initial data X, we need to solve the fixed point problem $X = \Psi(\tau, X)$. To this goal, we denote $\tau = T + \overline{\tau}$ and $X = X_0 + \overline{X}$, and observe that

$$X_0 + \overline{X} = \Psi\left(T + \overline{\tau}, X_0 + \overline{X}\right).$$

Let

$$N(\overline{\tau}, \overline{X}) = X_0 + \overline{X} - \Psi\left(T + \overline{\tau}, X_0 + \overline{X}\right) \tag{26}$$

and

$$N(\overline{\tau}, \overline{X}) = \left(N_1(\overline{\tau}, \overline{X}), N_2(\overline{\tau}, \overline{X})\right).$$

We are then led to solve the equation $N(\overline{\tau}, \overline{X}) = 0$. One notes that

$$\begin{aligned} D_X N(0,(0,0)) &= I_2 - D_X \Psi(T, X_0) \\ &= \begin{pmatrix} 1-d_{11} & -d_{12} \\ 0 & 1-d_{22} \end{pmatrix} = \begin{pmatrix} a_0' & b_0' \\ 0 & d_0' \end{pmatrix}, \end{aligned} \tag{27}$$

where I_2 is the identity matrix of order 2. A necessary condition for the bifurcation of the nontrivial periodic solutions near the semi-trivial periodic solution $(x_1^*(t), 0)$ is then

$$\det\left[D_X N\left(0, (0,0)\right)\right] = 0.$$

Suppose now that the conditions of **Case 1** are satisfied, that is, $a_0' \neq 0$ and $d_0' = 0$. It is easily seen that

$$\dim\left(\mathrm{Ker}\left[D_X N\left(0,(0,0)\right)\right]\right) = 1,$$

and a basis in $\mathrm{Ker}\left[D_X N(0,(0,0))\right]$ is $\left(-b_0'/a_0',1\right)$. Then the equation $N(\bar{\tau},\bar{X})=0$ is equivalent to

$$\begin{cases} N_1\left(\bar{\tau},\alpha Y_0+zE_0\right)=0, \\ N_2\left(\bar{\tau},\alpha Y_0+zE_0\right)=0, \end{cases}$$

where

$$E_0=(1,0), \quad Y_0=\left(-b_0'/a_0',1\right), \tag{28}$$

and $\bar{X}=\alpha Y_0+zE_0=\left(\alpha\left(-b_0'/a'\right)+z,\alpha\right)$ represents the the direct sum decomposition of X using the projections onto $\mathrm{Ker}\left[D_X N(0,(0,0))\right]$ (the central manifold) and $\mathrm{Im}\left[D_X N(0,(0,0))\right]$ (Chow & Hale, 1982). Let

$$\begin{cases} \xi_1\left(\bar{\tau},\alpha,z\right)=N_1\left(\bar{\tau},\alpha Y_0+zE_0\right), \\ \xi_2\left(\bar{\tau},\alpha,z\right)=N_2\left(\bar{\tau},\alpha Y_0+zE_0\right). \end{cases} \tag{29}$$

We need now solve the following system

$$\begin{cases} \xi_1\left(\bar{\tau},\alpha,z\right)=0, \\ \xi_2\left(\bar{\tau},\alpha,z\right)=0. \end{cases}$$

Since

$$\frac{\partial \xi_1}{\partial z}(0,0,0)=\frac{\partial N_1}{\partial x_1}(0,(0,0))=a_0'\neq 0,$$

by applying the implicit function theorem, one may locally solve the equation $\xi_1\left(\bar{\tau},\alpha,z\right)=0$ near $(0,0,0)$ with respect to z as a function of $\bar{\tau}$ and α and find $z=z(\bar{\tau},\alpha)$ such that $z(0,0)=0$ and

$$\xi_1\left(\bar{\tau},\alpha,z(\bar{\tau},\alpha)\right)=N_1\left(\bar{\tau},\alpha Y_0+z(\bar{\tau},\alpha)E_0\right)=0.$$

Taking the derivative of the implicit function defined above with respect to α at $(0,0)$, we may then deduce that

$$\frac{\partial N_1}{\partial x_1}(0,(0,0))\left(\frac{\partial x_1}{\partial \alpha}(0,0)+\frac{\partial x_1}{\partial z}\frac{\partial z}{\partial \alpha}(0,0)\right)+\frac{\partial N_1}{\partial x_2}(0,(0,0))\frac{\partial x_2}{\partial \alpha}(0,0)=0,$$

and consequently

$$a_0'\left(-b_0'/a_0' + \frac{\partial z}{\partial \alpha}(0,0)\right) + b_0' = 0,$$

which implies that

$$\frac{\partial z}{\partial \alpha}(0,0) = 0. \tag{30}$$

It now remains to study the solvability of the equation

$$\xi_2(\bar{\tau}, \alpha, z(\bar{\tau}, \alpha)) = N_2(\bar{\tau}, \alpha Y_0 + z(\bar{\tau}, \alpha) E_0) = 0. \tag{31}$$

The eq. (31) is called the determining equation and the number of its solutions equals the number of periodic solutions of (31) (Chow and Hale, 1982). In the following we shall proceed to solving (31) by using Taylor expansions. We denote

$$g(\bar{\tau}, \alpha) = \xi_2(\bar{\tau}, \alpha, z(\bar{\tau}, \alpha)). \tag{32}$$

First, we observe that

$$g(0,0) = N_2(0,(0,0)) = 0.$$

Second, we focus on the first order partial derivatives of g at (0,0). By (26) and (31) together with (27), it is easily seen that

$$\begin{aligned}\frac{\partial g}{\partial \alpha}(0,0) &= 1 - p_2 \frac{\partial \Phi_2}{\partial x_2}\left((1-\tilde{l})T; \Theta_1\left(\Phi(\tilde{l}T; X_0)\right)\right)\frac{\partial \Phi_2}{\partial x_2}\left(\tilde{l}T; X_0\right) \\ &= d_0' = 0.\end{aligned} \tag{33}$$

Since (30) holds and

$$\frac{\partial \Phi_2}{\partial x_1}\left((1-\tilde{l})T; \Theta_1\left(\Phi(\tilde{l}T; X_0)\right)\right) = 0, \tag{34}$$

$$\frac{\partial \Phi_2}{\partial \tau}\left((1-\tilde{l})T; \Theta_1\left(\Phi(\tilde{l}T; X_0)\right)\right) = 0, \tag{35}$$

it follows by a computational argument related to those employed in Georgescu *et al.* (2008), Appendixes *A-C*, that

$$\frac{\partial g}{\partial \bar{\tau}}(0,0) = 0. \tag{36}$$

Third, we compute the second order partial derivatives $\frac{\partial^2 g}{\partial \alpha^2}$, $\frac{\partial^2 g}{\partial \alpha \partial \bar{\tau}}$, $\frac{\partial^2 g}{\partial \bar{\tau}^2}$. After certain computations (see also Georgescu *et al.*, 2008, Appendixes *D*-*E*, for related results), we find that

$$\frac{\partial^2 g}{\partial \bar{\tau}^2} = 0, \tag{37}$$

the signs of $\frac{\partial^2 g}{\partial \alpha^2}$ and $\frac{\partial^2 g}{\partial \alpha \partial \bar{\tau}}$ being uncertain in our general settings. By constructing the second order Taylor expansion of g near $(0,0)$, one obtains from (33)-(37) that

$$g(\bar{\tau}, \alpha) = \frac{\partial^2 g}{\partial \alpha \partial \bar{\tau}}(0,0)\alpha\bar{\tau} + \frac{1}{2}\frac{\partial^2 g}{\partial \alpha^2}(0,0)\alpha^2 + o(\bar{\tau}, \alpha)(\bar{\tau}^2 + \alpha^2).$$

Let us denote

$$A = \frac{\partial^2 g}{\partial \alpha \partial \bar{\tau}}(0,0), \quad B = \frac{\partial^2 g}{\partial \alpha^2}(0,0)$$

and let $\alpha = k\bar{\tau}$, $k = k(\bar{\tau})$. It follows from the above equation that

$$g(\bar{\tau}, \alpha) = \bar{\tau}^2 \left[Ak + \frac{1}{2}Bk^2 + o(\bar{\tau}, \alpha)(1 + k^2) \right].$$

In conclusion, the above analysis may be summarized in the following result.

Theorem 10. *Assume that conditions* **(H1)-(H3)** *and* (11) *hold, together with those of* ***Case 1***. *Then the system (2) undergoes a supercritical bifurcation of nontrivial solutions if AB* < 0 *and a subcritical one if AB* > 0. *The case in which AB* = 0 *remains undetermined.*

Remark 11. *The final part of the existence argument can also be obtained by using the substitution* $\bar{\tau} = k\alpha$, $k = k(\alpha)$. *The signs of A and B, and therefore the conclusion of Theorem 10, depend heavily on the particulars of the system (2). Note that Theorem 1 of Georgescu et al.* (*2008*), *the counterpart of our Theorem 10, can be stated in a more explicite and narrower form* (*particularly, one may explicitly determine the signs of* A *and* B *therein*) *due to the fact that the system which is considered in Georgescu et al.* (*2008*) *has a much simpler form. This particular form leads to appropriate expressions* *for the partial derivatives of* Φ_1 *and* Φ_2 *, which in turn lead to simpler expressions for* A *and* B *. A similar analysis can be performed in the situation in which the condition of* ***Case 2*** *is satisfied, replacing only* E_0 *and* Y_0 *given by* (28) *with* $E_0 = (0,1)$ *and* $Y_0 = (1,0)$.

5. Conclusions

In this paper, we have proposed and analyzed a Cohen-Grossberg neural network composed of two neurons which are subject to non-simultaneous impulsive perturbations. By using Mawhin's continuation theorem, we first obtained sufficient conditions for the existence of semi-trivial periodic solutions. Subsequently, their stability has been investigated by using Floquet theory. Finally, we used a projection method introduced in Lakmeche and Arino (2000) and used also in Georgescu *et al.* (2008), and Zhang *et al.* (2008), to study the bifurcation of nontrivial periodic solutions. In this regard, we determined that when the relative magnitude of the impulse p_i passes a critical value, the semi-trivial periodic solution loses its stability and a bifurcation occurs. Actually, lower bounds on p_i's are required for the existence of semi-trivial periodic solutions and upper bounds on p_i's are required for their stability. It should be noted that the periodically forced oscillatory behavior of the neural networks is of great interest in many applications, being able to serve as a base model for the investigation of the control of the vital functions which occur with great regularity, such as heartbeat and respiratory movements.

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OSCILAȚII PERIODICE ŞI ANALIZA BIFURCAȚIILOR PENTRU O REȚEA NEURONALĂ DE TIP COHEN-GROSSBERG SUPUSĂ LA PERTURBAȚII DE TIP IMPULSIV

(Rezumat)

Articolul studiază comportamentul unei rețele neuronale de tip Cohen-Grossberg compusă din doi neuroni care sunt supuşi la perturbări de tip impulsive şi periodic. Mai întâi, utilizând teorema de prelungibilitate a lui Mawhin, sunt determinate condiții suficiente pentru existența soluțiilor periodice semi-triviale. Stabilitatea aimptotică a acestor soluții este mai apoi investigată utilizând teoria Floquet pentru ecuații diferențiale de tip impulsiv. În final, este discutată bifurcația unor soluții periodice netriviale prin intermediul unei metode de proiecție.