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ORIGINAL RESEARCH

Cigarette smoking on college campuses: an epidemical modelling approach

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Abstract
Cigarette smoking on college campuses has become a significant public health issue, which in turn led to an increasing focus on establishing programs to reduce its prevalence. In this paper, a compartmental model depicting the spread and cessation of the smoking habit on college campuses, obtained using theoretical principles often employed in mathematical epidemiology, is proposed and analysed. The existence and stability of the habitual smoking-free and habitual smoking-persistent equilibria, respectively, are explored in terms of a threshold parameter, hereby called the smokers generation number and denoted by R_c. A sensitivity analysis indicates that R_c is the most sensitive to the contact rate between habitual-smokers and occasional-smokers and to the rate of successfully quitting smoking. Numerical simulations of the proposed optimal control strategies reveal that the most effective approach to reduce the prevalence of cigarette smoking and possibly achieve a smoking-free campus should combine both control measures, namely allocating mandatory smoking rooms together with educating the public on the harmful effects of smoking and providing large scale guidance, counselling and support therapy to help students quit smoking.

Keywords Compartmental model · Stability results · Threshold parameter · Sensitivity indices · Optimal control

Mathematics Subject Classification 34D20 · 91D30 · 92D30

1 Introduction

The tobacco epidemic, one of the most significant public health issues the world has ever faced, causes the death of about six million individuals a year, more than five million of those deaths being due to first-hand smoking [1,3]. There are more than 7000 chemical substances in tobacco smoke, out of which at least 250 are identified

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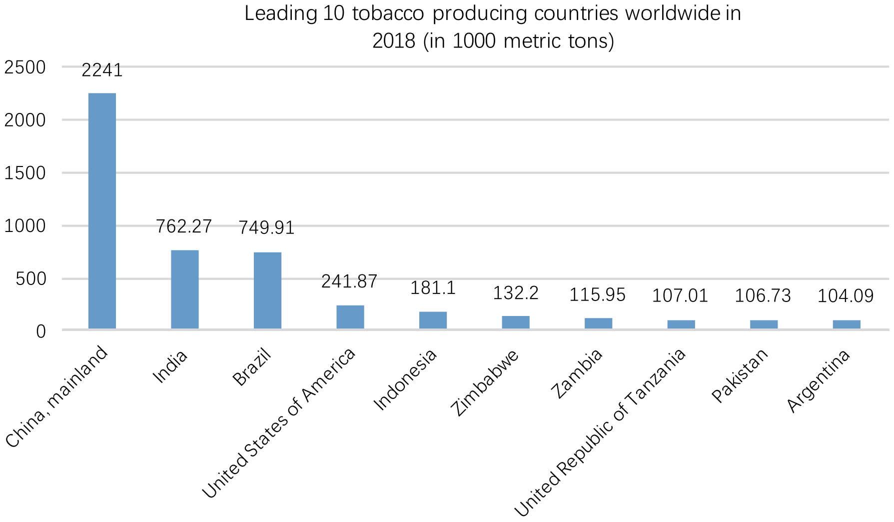


Fig. 1 The top ten leading producers of tobacco in 2016

to be unsafe and at least 70 are known to cause cancer [2]. Cigarette smoking actively harms the entire human body, reducing its overall health and causing heart and lung diseases, including several types of cancer [2].

The top ten worldwide leading tobacco production countries in 2016 are shown in Fig. 1. Due to a suitable climate for the cultivation of tobacco plants, China, India and Brazil were the leading producers, followed by the United States [4]. That year, China produced an amount of 2.8 million metric tons of tobacco, larger than the total output of the rest of the top ten producers combined.

The percentage of smokers among university students has varied significantly over the years. Studies published in 1993 and 1997 assessed that the percentages of the United States college students who smoked were 22% and 28% respectively [5]. According to a China Daily report, the Chinese Association on Tobacco Control (CATC) estimated that 22.5% of students spread across 11 provinces in China had tried smoking a cigarette [6].

In China, Xinhua news agency reported that Chinese institutions of higher education have a massive task to accomplish in order to make their school environments smoke-free, as the latest reports show that 98% of them have failed in tobacco control [7]. According to a report from the CATC, only 16 out of 800 colleges and universities surveyed over six months had passed the association's assessment and qualified as smoke-free campuses [6]. The survey also showed that only 4.2% of these institutions carried out promotional actions on tobacco control on their campuses, while cigarette butts were found on 73% of the surveyed campuses, especially in male dormitories [6].

In the United States, Harvard School of Public Health surveyed more than 14,000 students at 119 tertiary schools nationwide, asking them to detail on their lifetime tobacco use, and found that 23% of tertiary students said they had smoked cigarettes during the previous year. Also, 9% reported they were current cigarette users, while

3.7% said they were using chewing tobacco and 1.2% said they were currently smoking pipes [8]. The main finding of the study was that the percentage of college students who used tobacco products at least once a month was about 33% [8].

Smoking is viewed by certain college students as a social activity and over half of the student smokers do not regard themselves as habitual smokers [9]. As a consequence, the smoking behaviour of college students is frequently labelled as social smoking, defined in several ways: (a) young adult non-daily smoking that frequently happens in bars, restaurants, and nightclubs [10], (b) adolescent non-daily smoking that occurs only with other smokers [11], and (c) adolescent smoking that frequently happens in the presence of other individuals rather than alone [12]. Most students who smoke but do not identify as casual smokers might consider themselves “party smokers” or “weekend smokers” [13], a possible cause being the fact that most college students plan to quit smoking by the time that they graduate.

Research showed that people who quit smoking before the age of 30 eradicate the increased danger of smoking-related death almost entirely [14]. Creating a smoke-free environment for college students is then much more than an initiative to appease non-smokers who cannot withstand the scent of cigarette smoke, being about preserving and protecting the health of younger adults. To this goal, more and more campuses provide additional resources to help students kick their smoking addiction.

Mathematical modelling has become a vital tool in order to formulate and analyse preventive and control measures for reducing the spread of infectious diseases and curbing undesirable social behaviour [15–19]. The paper [20] proposed a simplified model for giving up smoking considering that a constant total population is divided into three classes: potential smokers (P), smokers (S) and former smokers who have quitted smoking permanently (Q). Later, another study [21] refined this model by introducing a new class Q_t, which represents smokers who temporarily quit smoking, and obtained a global stability result expressed in terms of a certain threshold, called the smokers generation number. The global stability of the unique smoking-persistent equilibrium of the model developed in [21] has been further analysed in [22]. Also, [23] derived and analysed a smoking model which accounts for an occasional smokers compartment, later extending the model to consider the possibility of quitters relapsing and becoming smokers again [24]. In [25], the model has been adapted to use fractional derivatives. Also, [26] used a multilevel mediation model for investigating the relationship between school safety, school satisfaction, and students’ cigarette smoking. Other studies on cigarette smoking have used statistical methods to analyse available quantitative data regarding the smoking habits of high school and university students [27–30].

Comparatively few studies have used compartmental ODE models to predict the prevalence of cigarette smoking among college students. In this paper, a mathematical model accounting for differences in the smoking habits of college students, from occasional-smokers to habitual-smokers, is formulated and investigated. We analyse the stability of the equilibria and identify the most sensitive parameters that lead to an increased on-campus prevalence of cigarette smoking among university students. Optimal control theory is then applied to find an appropriate control strategy to reduce the prevalence of cigarette smoking. The main goals of this paper are as follows.

1. To formulate, test and analyse a mathematical model for the on-campus prevalence of cigarette smoking among university students.
2. To analyse the most sensitive parameters that lead to an increased prevalence.
3. To determine appropriate control strategies to reduce the prevalence of cigarette smoking and possibly lead to a smoking-free campus.

Firstly, we formulate and analyse the model without control measures and investigate its stability properties in terms of a threshold parameter, the smokers generation number, determining also its sensitivity with respect to the model parameters. Interpreting the results of the sensitivity analysis, we are led to consider two control measures to steer the behaviour of the model. Optimal control theory is subsequently used to investigate the effectiveness of the proposed control measures, namely allocating mandatory rooms for smoking cigarettes on campus to prevent smokers from coming into contact with both the quit-smokers and occasional-smokers population when smoking cigarettes and educating the public on the harmful effects of smoking cigarettes, and providing large scale guidance, counselling and support therapy to help students quit smoking.

The remaining part of this paper is organised as follows. In Sect. 2, we introduce the compartmental ODE model of concern, stating the relevant biological assumptions together with the definitions of the various parameters employed. In Sect. 3, we determine the explicit expressions of the habitual smoking-free and the habitual smoking-persistent equilibria and characterize their stability. We also find the explicit expression of the smokers generation number and perform a sensitivity analysis with respect to the parameters of the model. In Sect. 4, we state the control problem and introduce the objective functional to be minimised, the Pontryagin's maximum principle being then applied to find necessary conditions for the existence of the optimal control. In Sect. 5, the parameter values are estimated by using survey data collected from over 1200 university students. In Sect. 6, we complement the previous theoretical analysis with several numerical simulations. Finally, Sect. 7 is concerned with a discussion of the main results and findings. Since cigarette smoking is a significant public health issue worldwide, we hope that the findings of this paper will be valuable for policy review in order to reduce cigarette smoking among university students.

2 Model formulation

The model formulated in this paper is related to the ones previously employed to discuss diverse social problems [18,19,25]. The model relies upon the following assumptions.

2.1 Assumptions of the model

1. Students within each compartment behave similarly.
2. The occasional-smoker population is made up of individuals who smoke cigarettes occasionally but are not addicted.

3. Some of the quit-smokers can be triggered to relapse and return to the occasional-smokers compartment.
4. The rate at which the habit of smoking is acquired is proportional to the product of the habitual-smokers and the occasional-smokers populations present.
5. The habit of smoking can be acquired on the university campus only.

We consider a compartmental model inspired by epidemiological considerations that captures the prevalence and dynamics of cigarette smoking among students on campus (CSASC). The student population on campus is divided into three compartments: the occasional-smokers population (O_s), the habitual-smokers population (S_s) and the quit-smokers population (Q_s). The occasional-smokers population is made up of students who have smoked cigarettes in their lifetime but are not addicted. The habitual-smokers population is made up of students who smoke regularly. The quit-smokers population is made up of both students who have quit smoking and students who have made up their mind not to smoke (non-smokers). All students on campus graduate from the university in one of the three states, as occasional-smokers, habitual-smokers or quit-smokers. It is considered that all individuals living on the university campus are automatically at risk of becoming habitual-smokers or occasional-smokers.

New students are admitted into the university campus at the beginning of every academic year at a rate Λ, out of which $\Lambda\delta$ are occasional-smokers and $\Lambda(1-\delta)$ of them have already made up their mind not to smoke or have quit smoking cigarettes, δ being then the proportion of the admitted students who are occasional-smokers. The occasional-smokers move out of the occasional-smokers compartment through graduation at a rate g or through joining the habitual-smokers population at a rate β. The students who are habitual smokers move out of the habitual-smokers compartment through graduation at a rate g or through quitting smoking, in which case they move to the quit-smokers compartment at a rate γ. The newly admitted students that have made up their minds to never smoke move directly into the quit-smokers compartment at a rate of $\Lambda(1-\delta)$. The students in the quit-smokers compartment move out of this compartment through graduation still as quit-smokers at a rate g or return to the occasional-smokers compartment at a relapse rate τ under the influence of their smoker friends.

The flow diagram is shown in Fig. 2, the description of the involved parameters being given in Table 1, together with their estimated values. The resulting compartmental ODE model is indicated below.

$$
\begin{aligned}
\frac{dO_s}{dt} &= \Lambda\delta - \beta O_s S_s - gO_s + \tau Q_s, \\
\frac{dS_s}{dt} &= \beta O_s S_s - (\gamma + g)S_s, \\
\frac{dQ_s}{dt} &= \Lambda(1-\delta) + \gamma S_s - (\tau + g)Q_s.
\end{aligned}
\tag{1}
$$

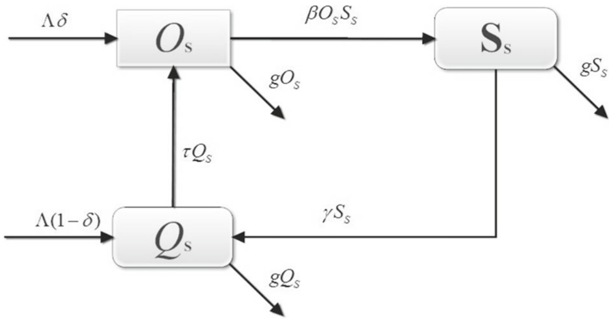


Fig. 2 A conceptual diagram of the mathematical model of cigarette smoking among students on campus

Table 1 Description of variables and parameters of the CSASC model

Description	Notation	Estimated value	Range explored	Unit
Occasional-smoker population	O_s			
Habitual-smoker population	S_s			
Quit-smoker population	Q_s			
Students admission rate	Λ	8000	1500 − 9500	year^{-1}
Probability that a freshman is an occasional-smoker	δ	0.3997	0.1–0.5	–
Contact rate between occasional-smokers and habitual-smokers	β	0.3525	0.2–0.45	day^{-1}
Quitting rate	γ	0.2218	0.1–0.4	day^{-1}
Relapse rate of quit-smokers	τ	0.0493	0.01–0.53	day^{-1}
Graduation rate	g	0.000684	0.000457 − 0.00721	–
Spontaneous recovery rate	b	0.26	0–1	day^{-1}

Similar values of the model parameters are presented in [21,23–25] ("–" to be understood as "dimensionless")

3 Model analysis

3.1 The invariant region

Let $N(t) = O_s(t) + S_s(t) + Q_s(t)$. It follows from (1) that $dN/dt = \Lambda - gN$, which implies that $\lim\sup_{t\to\infty} N(t) \leq \frac{\Lambda}{g}$. The feasible region is then

$$\mathcal{D} = \left\{(O_s(t), S_s(t), Q_s(t)) \in R_+^3 : N(t) \leq \frac{\Lambda}{g}\right\},$$

which is positively invariant. This means that in this region the system (1) is biologically meaningful and mathematically well posed, all solutions of (1) starting with initial data $(O_s(0), S_s(0), Q_s(0))$ in $\mathcal{D}$ remaining in $\mathcal{D}$ for all $t > 0$. See "Appendix A1" for further details.

3.2 The habitual smoking-free equilibrium

At the habitual smoking-free equilibrium, we assume that there is no habitual smoker left on campus. By solving the equilibrium system derived from (1), we obtain the explicit expression of the habitual smoking-free equilibrium point

$$E^0 = \left(O_s^0, S_s^0, Q_s^0\right) = \left(\frac{\Lambda(g\delta+\tau)}{g(g+\tau)}, 0, \frac{\Lambda(1-\delta)}{g+\tau}\right).$$

To further determine whether or not habitual smoking will persist on campus, we compute a relevant parameter which is similar in its scope to the basic reproduction number of mathematical epidemiology, hereby called the smokers generation number, and investigate the stability of the habitual smoking-free equilibrium in terms of this parameter.

3.3 The smokers generation number

The smokers generation number R_c, defined as the average number of secondary cases produced by a single habitual-smoker during his or her studies in an otherwise wholly occasional-smokers and quit-smokers population, is a key parameter which assesses the transmission of smoking habit to occasional-smokers and quit-smokers. For the CSASC model, the smokers generation number is obtained by using the next generation method [31]. It then follows (see "Appendix A2" for details) that the smokers generation number for the system (1) is given by

$$R_c = \frac{\beta\Lambda(g\delta+\tau)}{g(\gamma+g)(g+\tau)}. \tag{2}$$

3.3.1 Stability of the habitual smoking-free equilibrium

Theorem 1 *The habitual smoking-free equilibrium E^0 is locally stable if $R_c < 1$ and unstable if $R_c > 1$.*

Proof The Jacobian matrix associated to the system (1), evaluated at the habitual smoking-free equilibrium, is

$$J(O_s^0, S_s^0, Q_s^0) = \begin{bmatrix} -g & -\frac{\beta\Lambda(g\delta+\tau)}{g(g+\tau)} & \tau \\ 0 & \frac{\beta\Lambda(g\delta+\tau)}{g(g+\tau)} - (\gamma+g) & 0 \\ 0 & \gamma & -(g+\tau) \end{bmatrix}.$$

Its eigenvalues are

$$\lambda_1 = -g,$$
$$\lambda_2 = -(g+\tau),$$
$$\lambda_3 = (g+\tau)(R_c - 1).$$

It is clear that $\lambda_1 < 0$, $\lambda_2 < 0$ and $\lambda_3 < 0$ provided that $R_c < 1$, while $\lambda_3 > 0$ if $R_c > 1$. Consequently, the habitual smoking-free equilibrium is stable if $R_c < 1$ and unstable if $R_c > 1$. This completes the proof. □

3.4 Stability of the habitual smoking-persistent equilibrium

At the habitual smoking-persistent equilibrium, we assume that there are habitual smokers present on the campus. The habitual smoking-persistent equilibrium

$$E^* = (O_s^*, S_s^*, Q_s^*)$$

is obtained by setting the right-hand sides of (1) to zero and solving the associated equilibrium system. It is seen that

$$O_s^* = \frac{g+\gamma}{\beta}, \tag{3}$$

$$S_s^* = \frac{(g+\tau)(\gamma+g)(R_c-1)}{\beta(g+\tau+\gamma)}, \tag{4}$$

$$Q_s^* = \frac{\beta\Lambda\Big(g(1-\delta)+\gamma\Big) - g\gamma(g+\gamma)}{g\beta(g+\tau+\gamma)}. \tag{5}$$

Theorem 2 *The habitual smoking-persistent equilibrium E^* is locally stable if $R_c > 1$ and unstable if $R_c > 1$.*

Proof The Jacobian matrix associated to the system (1), evaluated at the habitual smoking-persistent equilibrium, is

$$J(O_s^*, S_s^*, Q_s^*) = \begin{bmatrix} -\dfrac{(g+\tau)(\gamma+g)(R_c-1)}{g+\tau+\gamma} - g & -(\gamma+g) & \tau \\ \dfrac{(g+\tau)(\gamma+g)(R_c-1)}{g+\tau+\gamma} & 0 & 0 \\ 0 & \gamma & -(g+\tau) \end{bmatrix}.$$

Solving for the eigenvalues of the Jacobian matrix at the habitual smoking-persistent equilibrium, we obtain the characteristic polynomial

$$p(\lambda) = \lambda^3 + A_2\lambda^2 + A_1\lambda + A_0,$$

in which

$$A_0 = (R_c - 1)g(\gamma + g)(g + \tau), \tag{6}$$

$$A_1 = \frac{(g+\tau)(g^2 + (\tau+\gamma)g + \gamma(\tau+\gamma))(R_c - 1)}{g+\tau+\gamma} + \frac{(g+\tau)(g^2 + (\tau+\gamma)g)}{g+\tau+\gamma}, \tag{7}$$

$$A_2 = \frac{(g^2 + (\tau+\gamma)g + \gamma\tau)(R_c - 1)}{g+\tau+\gamma} + \frac{g^2 + (\tau+\gamma)g + \tau(\tau+\gamma)}{g+\tau\gamma}. \tag{8}$$

From the Routh-Hurwitz criterion, E^* is stable if

$$A_0 > 0, A_2 > 0 \quad \text{and} \quad A_1 A_2 > A_0. \qquad \square$$

In view of (6) and (8), $A_0 > 0$ and $A_2 > 0$ if $R_c > 1$, and the first two Routh-Hurwitz conditions hold. We can further show (see "Appendix A3" for further details) that $A_1 A_2 > A_0$ for all parameter values such that $R_c > 1$. Therefore, the third Routh-Hurwitz condition also holds. Hence the habitual smoking-persistent equilibrium E^* is stable when $R_c > 1$ by the Routh-Hurwitz criterion.

3.5 Sensitivity analysis

To further understand the model dynamics and determine the robustness of model predictions with respect to parameter values, we investigate the sensitivity of the smokers generation number R_c with respect to the model parameters. Generally, sensitivity indices measure the relative changes in a state variable when a parameter changes. Particularly, the normalized forward sensitivity index (NFSI) of a variable with respect to a parameter is the relative change in that variable to the relative change in a given parameter. See also [32,33] for further details.

Definition 3.1 The normalized forward sensitivity index of a variable W that depends differentially on a parameter z is defined as

$$\frac{\partial W}{\partial z} \times \frac{z}{W} \tag{9}$$

We now compute the sensitivity indices of the smokers generation number R_c with respect to the parameters of the model. These indices indicate how sensitive R_c is to a change in each parameter. A positive sensitivity index indicates that an increase (decrease) in the parameter value leads to an increase (decrease) in the value of R_c whereas a negative sensitivity index signifies that an increase (decrease) in the parameter value leads to a decrease (increase) in the value of R_c.

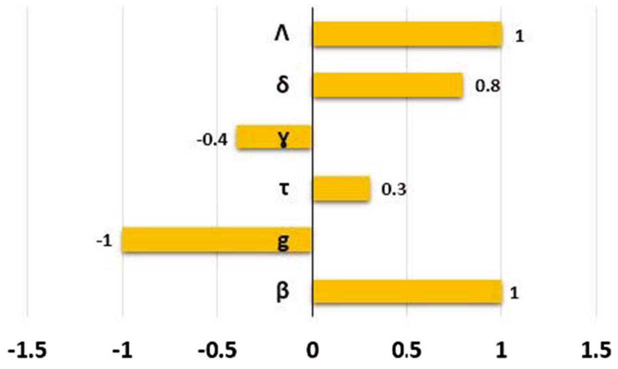


Fig. 3 Sensitivity indices for R_c with respect to the model parameters

Using the explicit expression of R_c given by (2) and Definition 3.1, the sensitivity indices of R_c with respect to each of its parameters are determined to be

$$
\begin{aligned}
\Upsilon_{\beta}^{R_c} &= 1, \\
\Upsilon_{\Lambda}^{R_c} &= 1, \\
\Upsilon_{\delta}^{R_c} &= \frac{\delta g}{\delta g + \tau}, \\
\Upsilon_{\gamma}^{R_c} &= -\frac{\gamma}{\gamma + g}, \\
\Upsilon_{\tau}^{R_c} &= -\frac{g\tau(\delta - 1)}{(g + \tau)(\delta g + \tau)}, \\
\Upsilon_{g}^{R_c} &= -\frac{2g^3\delta + g^2(\gamma\delta + (\delta + 3)\tau) + 2\tau g(\gamma + \tau) - \gamma\tau^2}{(\gamma + g)(\gamma + \tau)(\delta g + \tau)}.
\end{aligned}
$$

We now compute numeric values of the sensitivity indices and summarize our results in Fig. 3. This shows that R_c is most sensitive to variations in the values of β, Λ and g indicating that a 10% increase (decrease) in β or Λ will lead to a 10% increase (decrease) of the value of R_c. R_c is least sensitive to τ. Also, γ has a sensitivity index of -0.4, which indicates that 10% increase (decrease) in γ will lead to a 4% decrease (increase) of the value of R_c.

The smokers generation number has a positive correlation with β and Λ, which suggests that decreasing the contact rate β between occasional-smokers and habitual-smokers will help reducing the number of habitual smokers, which can be achieved through allocating mandatory on-campus rooms for cigarette smoking and enacting laws to ban smoking in public places on the campus. Also, the smokers generation number has a negative correlation with γ, which suggests that increasing the rate at which smokers quit smoking γ will decrease the prevalence of cigarette smoking on the campus. This may hopefully be achieved through advertising and advocating about the harmfulness of smoking cigarettes. A contour plot showing the dynamics of R_c in terms of γ, δ and τ is shown in Fig. 4.

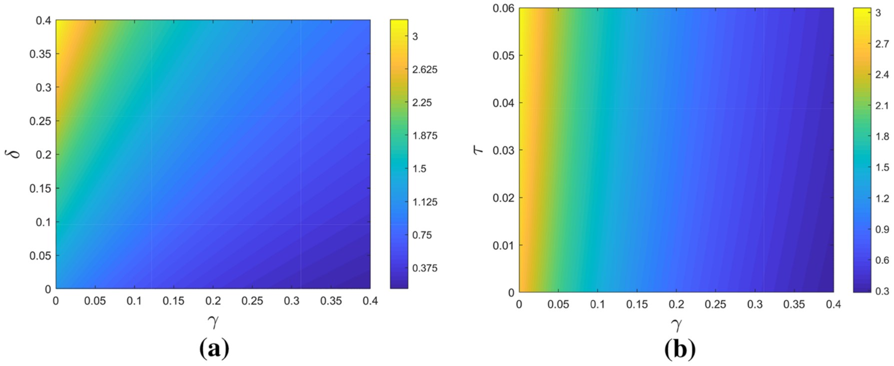


Fig. 4 The contour plot of R_c in terms of two of the three controllable parameters: γ (quitting rate of cigarette smokers), δ (admission rate of occasional smokers) and τ (relapse rate of quit-smokers)

4 Optimal control strategies

In this section, we augment our model (1) by introducing two time-dependent control measures, namely,

$u_1(t)$: allocating mandatory rooms for smoking cigarettes on campus to prevent habitual smokers from coming into contact with both the quit-smokers and occasional-smokers population when smoking cigarettes together with educating the public on the harmful effects of cigarette smoking

$u_2(t)$: providing large scale guidance, counselling and support therapy to help students quit smoking.

It is assumed that in the students population, the associated force of persuasion to smoke is reduced by a factor of $(1 - u_1(t))$ as more students gain access to education on the hazards of cigarette smoking. Furthermore, the habitual-smokers population is reduced by a factor of $(b + \gamma u_2(t))$ as more habitual smokers move into the quit-smokers compartment, where b is the spontaneous recovery rate.

The controlled version of the system (1) takes the form

$$\begin{aligned} \frac{dO_s}{dt} &= \Lambda\delta - (1 - u_1(t))\beta O_s S_s - gO_s + \tau Q_s, \\ \frac{dS_s}{dt} &= (1 - u_1(t))\beta O_s S_s - (b + \gamma u_2(t))S_s - gS_s, \\ \frac{dQ_s}{dt} &= \Lambda(1 - \delta) + (b + \gamma u_2(t))S_s - (\tau + g)Q_s, \end{aligned} \tag{10}$$

the objective functional being given by

$$J(u_1, u_2) = \int_0^T [c_1 S_s + c_2 u_1^2 + c_3 u_2^2]dt, \tag{11}$$

where T is the final time and the coefficients c_1, c_2, c_3 are positive weights. Our aim is to minimize the size of the habitual-smoker population while also minimizing the cost of using controls $u_1(t)$ and $u_2(t)$. Thus, we search for optimal controls u_1^* and u_2^* such that

$$J(u_1^*, u_2^*) = \min_{u_1,\, u_2} \{J(u_1, u_2) | u_1, u_2 \in \Omega\}, \tag{12}$$

where the control set is

$$\Omega = \{(u_1, u_2) | u_i :\ [0, T\ ] \to\ [0, \infty) \text{ Lebesgue measurable}, \quad i = 1, 2\}.$$

The term $c_1 S_s$ represents the cost of cigarette smoking prevalence, while $c_2 u_1^2$ is the cost of allocating mandatory rooms for cigarette smoking and of mass education on the hazards of cigarette smoking and $c_3 u_2^2$ is the cost of providing large scale guidance, counselling and support therapy to help students quit smoking. The necessary conditions that an optimal control must satisfy can be determined using Pontryagin's Maximum Principle [34]. This principle converts Eqs. (10) and (11) into a problem of point-wise minimizing the Hamiltonian H with respect to (u_1, u_2).

$$\begin{aligned} H = {} & c_1 S_s + c_2 u_1^2 + c_3 u_2^2 + \lambda_{O_s} \{\Lambda\delta - (1 - u_1(t))\beta O_s S_s - g O_s + \tau Q_s\}, \\ & + \lambda_{S_s} \{(1 - u_1(t))\beta O_s S_s - (b + \gamma u_2(t)) S_s - g S_s\}, \\ & + \lambda_{Q_s} \{\Lambda(1 - \delta) + (b + \gamma u_2(t)) S_s - (\tau + g) Q_s\}, \end{aligned}$$

where $\lambda_{O_s}, \lambda_{S_s}, \lambda_{Q_s}$ are the adjoint variables or co-state variables [34].

$$\begin{aligned} -\frac{d\lambda_{O_s}}{dt} &= \frac{\partial H}{\partial O_s} = -\ [(1 - u_1(t))\beta S_s + g\ ]\lambda_{O_s} +\ [(1 - u_1(t))\beta S_s\ ]\lambda_{S_s}, \\ -\frac{d\lambda_{S_s}}{dt} &= \frac{\partial H}{\partial S_s} = c_1 -\ [(1 - u_1(t))\beta O_s\ ]\lambda_{O_s} +\ [(1 - u_1(t))\beta O_s - (b + \gamma u_2(t)) - g\ ]\lambda_{S_s} \\ &\qquad +\ [(b + \gamma u_2(t))\ ]\lambda_{Q_s}, \\ -\frac{d\lambda_{Q_s}}{dt} &= \frac{\partial H}{\partial Q_s} = \tau\lambda_{O_s} -\ [(\tau - g)\ ]\lambda_{Q_s}. \end{aligned}$$

Since in our problem there are no terminal values for the state variables, we give transversality conditions at the final time T by

$$\lambda_{O_s}(T) = \lambda_{S_s}(T) = \lambda_{Q_s}(T) = 0.$$

On the interior of the control set, where $0 < u_i < 1$, for $i = 1, 2$, we have

$$\begin{aligned} \frac{\partial H}{\partial u_1} &= 2c_2 u_1 + \beta O_s S_s (\lambda_{O_s} - \lambda_{S_s}) = 0, \\ \frac{\partial H}{\partial u_2} &= 2c_3 u_2 + \gamma S_s (\lambda_{Q_s} - \lambda_{S_s}) = 0. \end{aligned}$$

Table 2 Summary of the data obtained from the cigarette-smoking survey ($N = 1220$)

Description	First year	Second year	Third year	Fourth year	Total
Male	243	103	163	136	645
Female	164	112	124	175	575
Indecisive students	104	64	111	107	386
Occasional smokers	75	33	24	14	146
Non-smokers	216	96	132	163	607
Quit smokers	12	22	20	27	81
Some friends smoke cigarettes daily					
Yes	124	105	148	138	515
No	176	107	98	101	482
N/A	107	3	41	72	223
Some friends smoke cigarettes occasionally					
Yes	79	65	56	64	264
No	187	129	118	164	598
N/A	141	21	113	83	358
Started smoking again (relapse smokers)	21	14	18	19	72

We then obtain that

$$u_1 = \frac{\beta O_s S_s (\lambda_{S_s} - \lambda_{O_s})}{2c_2},$$
$$u_2 = \frac{\gamma S_s (\lambda_{S_s} - \lambda_{Q_s})}{2c_3}.$$

By standard control arguments involving the bounds on the control variables, we conclude that

$$u_1^* = \min\left\{1, \max\left(0, \frac{\beta O_s S_s (\lambda_{S_s} - \lambda_{O_s})}{2c_2}\right)\right\},$$
$$u_2^* = \min\left\{1, \max\left(0, \frac{\gamma S_s (\lambda_{S_s} - \lambda_{Q_s})}{2c_3}\right)\right\}.$$

5 Estimation of parameters

A total of 1220 undergraduate students from various faculties at Jiangsu University were surveyed for this study. We used a purposive sampling technique to select these students so that we could get occasional-smokers, habitual-smokers and quit-smokers all in the same sample. The distribution of students within the compartments of the model is given in Table 2. In the sample, there were a total of 645 male students and 575 female students.

The outcome of the variable susceptibility to habitual smoking and occasional smoking among those who have never tried and those who have tried cigarettes was assessed asking students the following questions based on [35].

1. Do you think you might try smoking cigarettes in the future?
2. If one of your best friends was to offer you a cigarette would you smoke it?
3. Do you think you will smoke a cigarette at any time in the next year?

The answers to these three questions have four options each: (a) definitely yes, (b) probably yes, (c) probably not, and (d) definitely not. Students who answered "definitely not" to all three questions were considered non-susceptible to cigarette smoking; otherwise, they were considered susceptible or occasional smokers. A similar approach was used to construct the remaining questions of the survey.

The estimates for the model parameters are obtained using data from the survey, following methods similar to those previously used in the available literature [18,33]. This provides crude estimations in order to demonstrate the potential applicability of the model. Conversely, there are some restrictions in computing our approximations. Some of the parameters such as Λ, δ, β, γ and g are to be computed using continuous data on contacts. Since we only have data from a single time point, we considered a simple, improvised method to provide crude estimations for these parameters.

Estimate of Λ:
On average, 8000 first-year students are admitted to Jiangsu University every year. Therefore, the admission rate of students per day is computed as 8000/(365.25), which is 21.903 per day. We assume that the number of students admitted could be as low as 1500 and as high as 9500 for some particular years. These low and high admission numbers are used to compute the range explored $(4.11 - 26.01)$ for the admission rate.

Estimate of δ:
From the survey results in Table 2, 146 students are occasional cigarette smokers before they were admitted into the university. The rate at which a student admitted is an occasional cigarette smoker is computed as (146/365.25) which is equal to 0.3997 per day.

Estimate of γ:
In the survey sample, 81 smokers have quit cigarette smoking. Therefore, the rate of quitting cigarette smoking is $(81) \times (1/365.25)$ which is equal to 0.2218.

Estimate of τ:
To compute the relapse rate of cigarette smoking, we used the number of non-smokers who have started smoking again, due to their association with new friends and student-smokers. In this regard, 72 out of the 777 non-smokers agreed to having started smoking again. This means that the rate at which a non-smoker will start smoking again is dependent on the average time of association with the friends who smoke cigarettes. The rate is then computed as $(72/365.25) \times (6/24)$ which is 0.0493 per day.

Estimate of g:
The length of the undergraduate program is four years, and students who fail to graduate within the stipulated period are given a maximum extension of two years. However, the

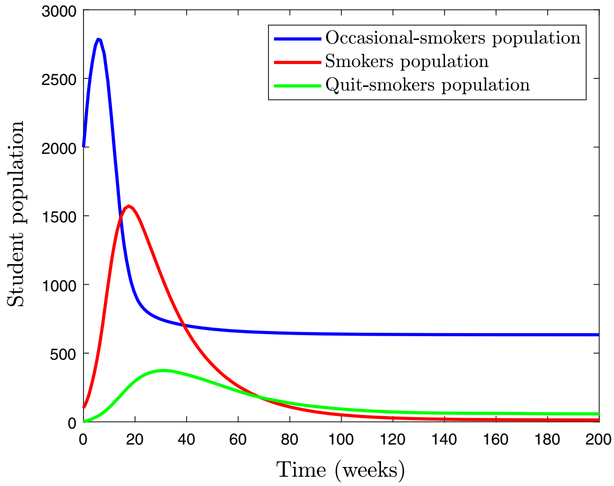


Fig. 5 Simulation showing the dynamics of occasional-smokers population, habitual-smokers population and quit-smokers population

majority of the students graduate within four years. This means that the average time required for graduation is four years, leading to $g = 1/(4 \times 365)$, which is 0.000684 per day.

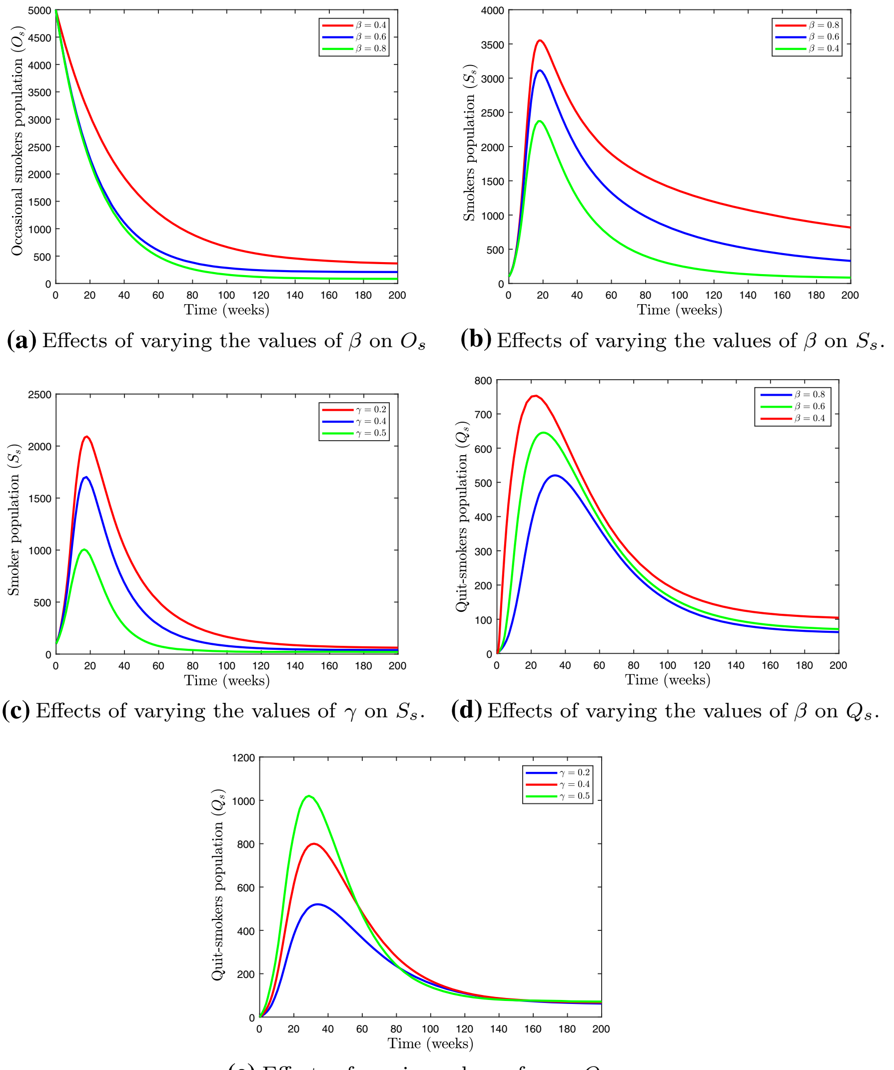
6 Numerical simulations

The behaviour of the model system (1) is numerically investigated using parameter values from Table 1 and realistic initial conditions. The numerical simulations are conducted using MATLAB, the results being given in the Figs. 5, 6, 7 and 8 in order to illustrate the behaviour of the model system for different values of the model parameters and the effects of the two optimal control measures on the habitual-smokers population. Figure 6a–e show the effects of the variations of β (the contact rate between habitual smokers and occasional smokers) and γ (quitting rate of cigarette smoking) on the compartment sizes, while keeping the remaining parameters in Table 1 unchanged.

Figure 6a shows the dynamics of the occasional-smokers population as the contact rate with habitual-smokers β increases. It is observed that the occasional-smokers population decreases as β rates increase, which is a result of occasional-smokers coming into more contacts with habitual-smokers (peer influence) that influences them to smoke habitually rather than occasionally.

Figure 6b indicates that an increase in the contact rate β increases the habitual-smoker population for a significant amount of time. As seen in Fig. 6c, the habitual-smoker population decreases as the quitting rate γ increases. As more habitual-smokers quit their smoking addiction, the habitual-smokers population will decrease, leading to an increase in the quit-smokers population.

Figure 6d indicates that the quit-smokers population decreases as a result of an increase in the contact rate β. The decrease in the quit-smokers population is due to habitual-smokers influencing some of the quit-smokers to relapse to cigarette smoking.



(a) Effects of varying the values of β on O_s

(b) Effects of varying the values of β on S_s.

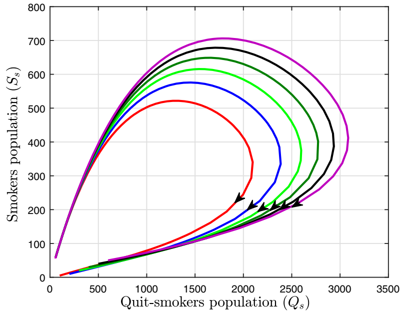
(c) Effects of varying the values of γ on S_s.

(d) Effects of varying the values of β on Q_s.

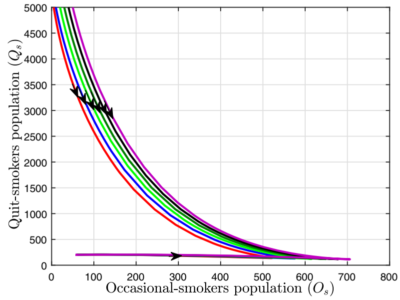
(e) Effects of varying values of γ on Q_s.

Fig. 6 Simulations showing changes in the sizes of **a** occasional-smokers population O_s for different values of γ, **b** habitual-smokers population S_s for different values of β, **c** habitual-smokers population S_s for different values of γ, **d** quit-smokers population Q_s for different values of β, **e** quit-smokers population Q_s for different values of γ

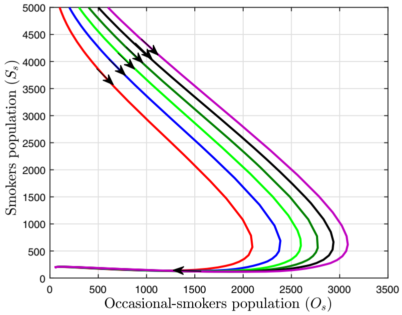
It is seen from Fig. 6e that the quit-smokers population increases as a result of an increase in the quitting rate γ, which is due to habitual-smokers quitting their cigarette addiction and joining the quit-smokers population. However, after a certain number of weeks, the quit-smokers population decreases to a lower peak due to some quit-smokers relapsing and returning to cigarette smoking. Figure 7a–c show the phase plane portraits of the occasional-smokers population, habitual-smokers population



(a) Phase plane portrait of habitual-smokers population against quit-smokers population.



(b) Phase plane portrait of quit-smokers population against occasional-smokers population.



(c) Phase plane portrait of habitual-smokers population against occasional-smokers population.

Fig. 7 Simulations showing the phase plane portraits of the different populations. The graphs show the phase plane portraits for the smoking-persistent model with the smokers generation number $R_c > 1$. They have been obtained by varying the initial conditions of O_s, S_s and Q_s, while keeping the other parameters fixed, as shown in Table 1

and quit-smokers population, illustrating the dynamics of the populations for different initial conditions when $R_c > 1$.

Furthermore, the effects of the optimal control strategies on the prevalence of cigarette smoking among on-campus students are also investigated. The optimal controls are determined by solving the optimality system, consisting of 6 ODEs and representing the state and adjoint equations. The numerical approach deals with a two-point boundary value problem with the boundary conditions at $t = 0$ and $t = T$ by using the MATLAB package bvp4c. Using the approach of employing only one control measure at a time and the approach which combines both control measures at the same time, we investigate and compare the corresponding results of the numerical simulations.

For the numerical simulations, we choose values for the variables and parameters based on the estimates from the survey, as shown in Table 1 above, for which the smokers generation number takes the value $R_c = 3.4256$. Also, to illustrate the effect of the optimal control strategies on the prevalence of cigarette smoking among the

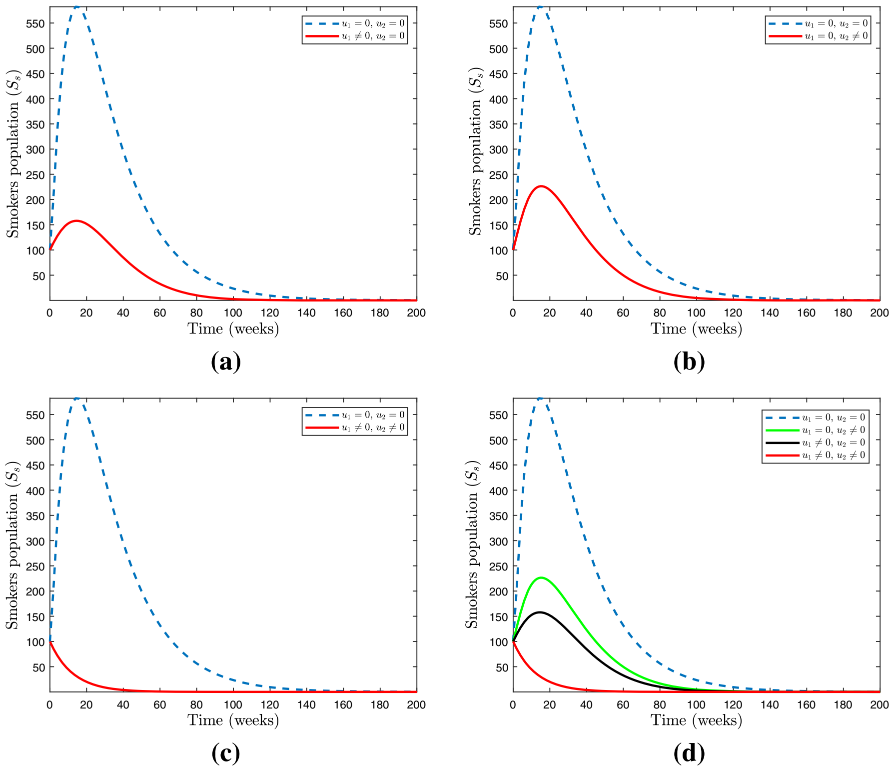


Fig. 8 Simulations showing the effects of the two control measures u_1 and u_2 on the sizes of the model populations

students population, we use the following weight factors: $c_1 = 65$, $c_2 = 92$ and $c_3 = 75$.

The optimal strategy integrates both control measures and applies them simultaneously to control the prevalence of cigarette smoking on campus. Figure 8a shows a substantial difference in the sizes of the habitual-smoker population with and without using the control measure u_1. Without using the control measures, the size of the habitual-smokers population increases faster and reaches a higher peak then it decreases slowly to the lower level, while in the controlled case the size of the habitual-smokers population increases slightly and reaches a lower peak as compared then it decreases faster to the lower level, which suggests that under the optimal control strategy u_1, the prevalence of cigarette smoking will be kept at low levels.

As shown in Fig. 8b, the size of the habitual-smokers population decreases rapidly under the optimal control strategy u_2, compared to what happens in the uncontrolled case, which suggests that under the optimal control strategy u_2, the prevalence of cigarette smoking among students will be kept at low levels. Figure 8c shows a substantial difference in the sizes of the habitual-smokers population with and without using the control measures u_1 and u_2. Without using the control measures, the size of the habitual-smokers population increases faster and reaches a higher peak then

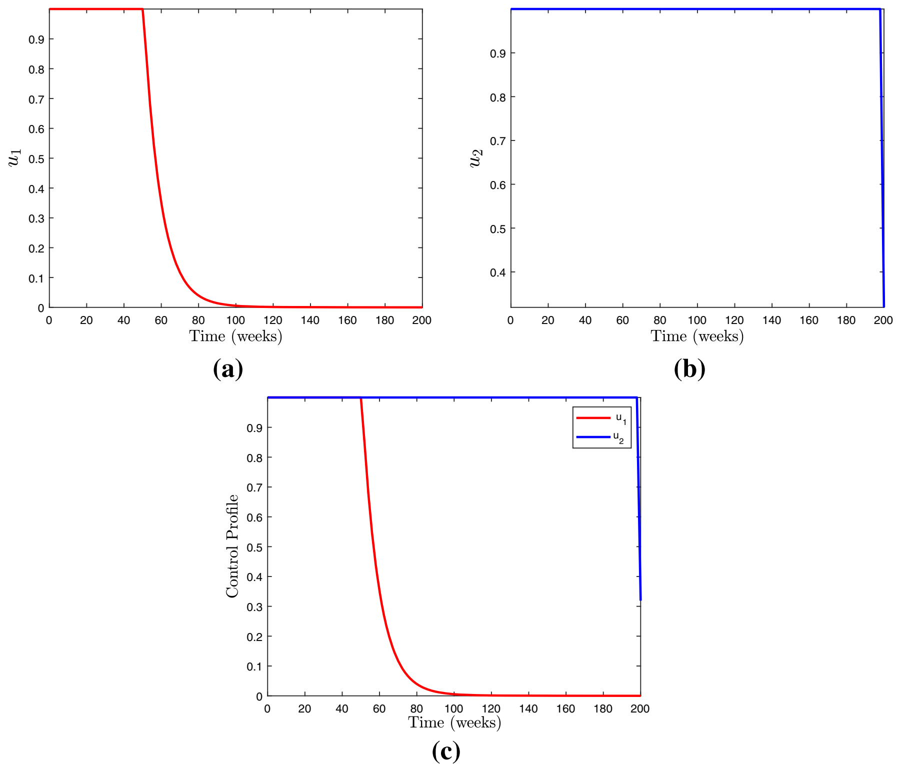


Fig. 9 Simulations showing the optimal control profiles of u_1 and u_2

it decreases slowly to the lower level, while in the controlled case the size of the habitual-smokers population decreases immediately to the lower level.

Figure 8d illustrates the simulations of the comparison of the effect of the different control strategies. The control profiles for u_1, u_2 and (u_1, u_2) are shown in Fig. 9a–c, respectively. It is observed from Fig. 9c that the control profile u_1 starts from the upper bound and gradually decreases to the lower level, which indicates that education on the negative effects of cigarette smoking and media advertisement about the harmful effects of cigarette smoking should be intensified at the start of every academic year. Also, the control profile u_2 stays at the upper bound for a relatively longer period compared to u_1, which indicates that the guidance, counselling and therapy services should continue throughout the academic year.

7 Discussion and conclusions

In this paper, a compartmental ODE model is developed to investigate the prevalence of cigarette smoking among university students on campus. The student population is divided into three compartments: occasional-smokers, habitual-smokers and quit-smokers. Theoretical principles from compartmental modelling of epidemic diseases are used to model the recruitment of students into the habitual-smokers compartment and investigate their possibilities to quit smoking. We assume that the recruitment of habitual-smokers is an imitation process, influenced by peer pressure.

First, the region where the model is feasible and mathematically well-posed is determined. A threshold parameter called the smokers generation number, similar in its scope to the basic reproduction number of epidemic models and denoted by R_c, is used to characterize the decrease and growth of the habitual-smokers population on the university campus, its explicit expression being obtained by means of the next generation matrix approach. The habitual smoking-free equilibrium is found to be stable if $R_c < 1$ and unstable if $R_c > 1$. Habitual-smokers may persist in the population when $R_c > 1$, in this case the model having a single habitual smoking-persistent equilibrium. Therefore, specific health and counselling programs should be established in that case to reduce the number of habitual-smokers on campus.

A sensitivity analysis is performed, the sensitivity indices of the smokers generation number R_c with respect to the model parameters being explicitly computed and then numerically estimated. Since R_c is a measure of the recruitment of new habitual-smokers, these sensitivity indices enable us to detect the relative significance of different parameters for the spread of cigarette smoking on campus. It is determined that R_c has a negative correlation with parameters g and γ and a positive correlation with β, δ and Λ.

From the numerical estimations of the sensitivity indices, it is seen that R_c is the most sensitive to the contact rate β and to the quitting rate γ. Therefore, we suggest that rules must be put in place to stop the students from smoking in public and mandatory rooms must be allocated for smoking on campus. The same numerical estimations also underline the significance of identifying contact processes between habitual-smokers and the other students, in order to impede the spread of cigarette smoking within the university campus via imitation mechanisms and peer pressure.

In order to assess the efficiency of certain control interventions on the spread of cigarette smoking, two time-dependent controls have been used to augment the model. It was observed that cigarette smoking could be eliminated if the control measures are implemented and strictly adhered to by all students on campus, from the beginning of the academic year. In addition, we investigated the role of the contact rate β together with other relevant parameters such as quitting rate γ and occasional-smokers admission rate δ through graphical plots. Since reliable data about cigarette smoking is not easily available in Asia, our mathematical model enables us to establish the extent of on-campus cigarette smoking and to propose measures to diminishing its prevalence based upon the sample data from our survey.

The model presented in this paper is theoretical, but backed with survey data. Its purpose is to simulate real-world situations and policies that can be of help to policy-makers in regulating cigarette smoking, that has plagued university campuses worldwide. As it often happens with model development, our model is not without limitations. Particularly, we assume the homogeneous mixing of the student populations under consideration, which is practically impossible to achieve in a university campus. Also, opinions on cigarette smoking may differ due to various religious, cultural, behavioural, social and environmental factors.

The initiation of cigarette smokers is presumed to be determined by imitation. Although this is the critical initiation mechanism, it is vital to consider self-initiation into cigarette smoking because of family smoking record, personal sentiments and self-pressure to fit into a group of friends with a social status. Notwithstanding these hindrances, the model presents a distinct endeavour to analyse and control the dynamics of cigarette smoking within a specific set of socially relevant real-world circumstances.

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Author contributions PH and ZH conceived the study, participated in model formulation and model analysis and helped drafting the intermediary versions of the manuscript. P.G. participated in model formulation and model analysis, helped drafting the intermediary versions of the manuscript and drafted the final version of the manuscript. PH and LZ carried out the stability analysis of the model steady states and helped drafting the intermediary versions of the manuscript. All authors read and approved the final manuscript.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

Data and code availability Procedures for the estimation of parameters have been stated in the body of the paper or referred to in the reference section. The graphs were produced using MATLAB version 2018a, available from https://www.mathworks.com/products/matlab.html.

Appendix

A1: Invariant region and positivity of solutions

Theorem 3 *If $O_s(0)$, $S_s(0)$ and $Q_s(0)$ are non-negative, then $O_s(t)$, $S_s(t)$ and $Q_s(t)$ are also non-negative for all $t > 0$. Moreover, the region $\mathcal{D}$ given by*

$$\mathcal{D} = \{(O_s(t), S_s(t), Q_s(t)) \in R^3_+ : O_s(t) + S_s(t) + Q_s(t) \leq \frac{\Lambda}{g}\}$$

is positively invariant.

Proof Let $(O_s(0), S_s(0), Q_s(0))$ be a set of non-negative initial conditions and denote by $[0, t_{\max})$ the maximum interval of existence of the corresponding solution. Let $t_1 = \sup\{0 < t < t_{\max} : O_s, S_s$ and Q_s are positive on $[0, t\ ]\}$. Since $O_s(0)$, $S_s(0)$, and $Q_s(0)$ are non-negative, then $t_1 > 0$.

If $t_1 < t_{\max}$ then, by using the variation of constants formula to the first equation of the system (1), we obtain

$$O_s(t_1) = \mathcal{U}(t_1, 0) O_s(0) + \int_0^{t_1} \Lambda \mathcal{U}(t_1, \omega) d\omega,$$

where

$$\mathcal{U}(t_1, \omega) = e^{\int_{\omega}^{t_1} (\beta+g)(O_s) dO_s}.$$

This implies that $O_s(t_1) > 0$. It can be shown in the same manner that this is the case for the other variables as well. This contradicts the fact that t_1 is the supremum because at least one of the variables should be equal to zero at t_1. Therefore $t_1 = t_{\max}$.

Let $N(t) = O_s(t) + S_s(t) + Q_s(t)$. Summing up all equations of (1), we have $\dfrac{dN(t)}{dt}(t) = \Lambda - gN(t)$. This implies

$$N(t) = \frac{\Lambda}{g} + \left(N(0) - \frac{\Lambda}{g} \right) e^{-gt}.$$

Then $N(t) \leq \dfrac{\Lambda}{g}$ if $N(0) \leq \dfrac{\Lambda}{g}$, which establishes the invariance of $\mathcal{D}$ as required. □

A2: The smokers generation number R_c

The smokers generation number for the system (1) is obtained using the next generation method. For this system, $\mathcal{F}$ and $\mathcal{V}$ are given by

$$\mathcal{F} = \left[\beta O_s S_s \right], \qquad \mathcal{V} = \left[(\gamma + g) S_s \right].$$

The associated Jacobian matrices of $\mathcal{F}$ and $\mathcal{V}$ at the smoking-free equilibrium are given by

$$F = \left[\frac{\beta\lambda(g\delta + \tau)}{g(g+\tau)} \right], \qquad V = \left[(\gamma + g) \right].$$

The smokers generation number R_c equals the spectral radius (dominant eigenvalue) of the matrix FV^{-1}, given by:

$$R_c = \frac{\beta\Lambda(g\delta + \tau)}{g(\gamma + g)(g + \tau)}. \tag{13}$$

A3: The stability of the habitual smoking-persistent equilibrium

Theorem 4 *The habitual smoking-persistent equilibrium E^* is locally asymptotically stable if $R_c > 1$ and unstable if $R_c < 1$.*

Proof The coordinates of the habitual smoking-persistent equilibrium E^*, obtained by solving the equilibrium relations associated to the system (1), are

$$O_s^* = \frac{g+\gamma}{\beta}, \tag{14}$$

$$S_s^* = \frac{(g+\tau)(\gamma+g)(R_c-1)}{\beta(g+\tau+\gamma)}, \tag{15}$$

$$Q_s^* = \frac{\beta\Lambda\Big(g(1-\delta)+\gamma\Big) - g\gamma(g+\gamma)}{g\beta(g+\tau+\gamma)}. \tag{16}$$

The Jacobian matrix of the system (1), evaluated at the habitual smoking-persistent equilibrium E^* is

$$J(O_s^*, S_s^*, Q_s^*) = \begin{bmatrix} -\frac{(g+\tau)(\gamma+g)(R_c-1)}{g+\tau+\gamma} - g & -(\gamma+g) & \tau \\ \frac{(g+\tau)(\gamma+g)(R_c-1)}{g+\tau+\gamma} & 0 & 0 \\ 0 & \gamma & -(g+\tau) \end{bmatrix}.$$

Solving for the eigenvalues of the Jacobian matrix at the habitual smoking-persistent equilibrium, we obtain the characteristic polynomial:

$$p(\lambda) = \lambda^3 + A_2\lambda^2 + A_1\lambda + A_0, \tag{17}$$

in which

$$A_0 = (R_c-1)g(\gamma+g)(g+\tau), \tag{18}$$

$$A_1 = \frac{(g+\tau)(g^2+(\tau+\gamma)g+\gamma(\tau+\gamma))(R_c-1)}{g+\tau+\gamma} + \frac{(g+\tau)(g^2+(\tau+\gamma)g)}{g+\tau+\gamma}, \tag{19}$$

$$A_2 = \frac{(g^2+(\tau+\gamma)g+\gamma\tau)(R_c-1)}{g+\tau+\gamma} + \frac{g^2+(\tau+\gamma)g+\tau(\tau+\gamma)}{g+\tau\gamma}. \tag{20}$$

We use the Routh - Hurwitz stability criterion to determine the stability of the characteristic polynomial equation associated to (17). From this criterion, if conditions

$$A_2 > 0, \quad A_0 > 0, \quad A_1A_2 > A_0$$

hold true, then all the roots of the characteristic polynomial equation have negative real parts which means that the habitual smoking-persistent equilibrium is stable.

From equations (18) and (20) above, the first two conditions are true for $R_c > 1$, as in this case A_2 and A_0 are both positive quantities. The third condition reduces to

$$\left[g^4 + (\gamma+\tau)g^3 + (\gamma+\tau)^2 g^2 + (\gamma+\tau)(\gamma\tau+\gamma^2)g + (\gamma+\tau)\tau\gamma^2\right](g+\tau)(R_c-1)^2$$
$$+\left[g^4 + (\gamma+\tau)g^3 + (\gamma\tau+\tau^2)g^2 + (\gamma+\tau)(\gamma\tau+\tau^2)g + (\gamma+\tau)^2\tau\gamma\right](g+\tau)(R_c-1)$$
$$+\left[g^4 + (\gamma+\tau)g^3 + (\gamma+\tau)^2 g^2 + (\gamma+\tau)(\gamma\tau+\tau^2)g\right](g+\tau) > 0$$

which holds true for all parameter values such that $R_c > 1$. Thus, the habitual smoking-persistent equilibrium is stable when $R_c > 1$ by the Routh - Hurwitz criterion. □

References

1. Laud, A.D.J., Marian, F.: The threats of globalization on global public health with specific reference to tobacco control. Public Health Res. **6**(5), 132–142 (2016). https://doi.org/10.5923/j.phr.20160605.02
2. U.S. Department of Health and Human Services: The Health Consequences of Smoking—50 Years of Progress: A Report of the Surgeon General. Atlanta: U.S. Department of Health and Human Services, Centers for Disease Control and Prevention, National Center for Chronic Disease Prevention and Health Promotion, Office on Smoking and Health (2014). (Accessed 21 Feb 2018)
3. World Health Organization: WHO Report on the Global Tobacco Epidemic. World Health Organization, Geneva (2011). (Accessed 22 Feb 2018)
4. World Tobacco Production By Country | Statista. https://www.statista.com/statistics/261173/leading-countries-in-tobacco-producti. (Accessed 2 Oct 2018)
5. The 14th World Conference on Tobacco or Health (WCTOH) (2014). http://www.who.int/tobacco/wctoh/en/
6. Chinese Students Begin To Smoke At A Younger Age (2011). http://www.chinatoday.com.cn/ctenglish/se/txt/2011-09/28/content_394307.htm. (Accessed 02 Oct 2018)
7. Most Chinese Colleges Fail In Tobacco Control. http://www.chinatoday.com.cn/ctenglish/se/txt/2011-09/28/content_394298.htm. (Accessed 02 Oct 2018)
8. Results from the 2013 National Survey on Drug Use and Health: Summary of National Findings. https://www.samhsa.gov/data/sites/default/files/NSDUHresultsPDFWHTML2013/Web/NSDUHresults2013.pdf
9. Levinson, A.H., Campo, S., Gascoigne, J., Jolly, O., Zakharyan, A., Tran, Z.V.: Smoking, but not smokers: identity among college students who smoke cigarettes. Nicotine Tob. Res. **9**(8), 845–852 (2007)
10. Philpot, S.J., Ryan, S.A., Torre, L.E., Wilcox, H.M., Jalleh, G., Jamrozik, K.: Effect of smoke-free policies on the behaviour of social smokers. Tob. Control **8**, 278–281 (1999)
11. Gilpin, E.A., White, V.M., Pierce, J.P.: How effective are tobacco industry bar and club marketing efforts in reaching young adults? Tob. Control **14**, 186–192 (2005)
12. Moran, S., Wechsler, H., Rigotti, N.A.: Social smoking among US college students. Pediatrics **114**, 1028–1034 (2004)
13. Colder, C.R., Lloyd-Richardson, E.E., Flaherty, B.P., Hedeker, D., Segawa, E., Flay, B.R.: The natural history of college smoking: trajectories of daily smoking during the freshman year. Addict. Behav. **31**, 2212–2222 (2006)
14. Smoke & Tobacco Free Campus: Supporting College Student Health. https://www.affordablecollegesonline.org/college-resource-center/tobacco-free-co. (Accessed 05 Oct 2018)
15. Zhang, H., Harvim, P., Georgescu, P.: Preventing the spread of schistosomiasis in Ghana: possible outcomes of integrated optimal control strategies. J. Biol. Syst. **25**(4), 625–655 (2017)
16. Okosun, O., Rachid, O., Nizar, M.: Optimal control strategies and cost-effectiveness analysis of a malaria model. BioSystems **111**, 83–101 (2013)
17. Castillo-Chavez, C., Song, B.: Dynamical models of tuberculosis and their applications. Math. Biosci. Eng. **1**(2), 361–404 (2004)

18. Amdouni, B., Paredes, M., Kribs-Zaleta, C., Mubayi, A.: Why do students quit school? Implications from a dynamical modelling study. Proc. R. Soc. A (2017). https://doi.org/10.1098/rspa.2016.0204
19. Nyabadza, F., Ogbogbo, C.P., Mushanyu, J.: Modelling the role of correctional services on gangs: insights through a mathematical model. R. Soc. Open Sci. **4**, 170511 (2017). https://doi.org/10.1098/rsos.170511
20. Castillo-Garsow, C., Jordan-Salivia, G., Herrera, A.R.: Mathematical Models for the Dynamics of Tobacco Use, Recovery, and Relapse, Technical Report Series BU-1505-M. Cornell University, Ithaca (1997)
21. Sharomi, O., Gumel, A.B.: Curtailing smoking dynamics: a mathematical modeling approach. Appl. Math. Comput. **195**(2), 475–499 (2008)
22. Lahrouz, A., Omari, L., Kiouach, D., Belmaâti, A.: Deterministic and stochastic stability of a mathematical model of smoking. Stat. Probab. Lett. **81**(8), 1276–1284 (2011)
23. Zaman, G.: Qualitative behavior of giving up smoking models. Bull. Malays. Math. Sci. Soc. **34**(2), 403–415 (2011)
24. Zaman, G.: Optimal campaign in the smoking dynamics. Comput. Math. Methods Med. 2011, Article ID 163834, 9 p (2011)
25. Ertürk, V.S., Zaman, G., Momani, S.: A numeric-analytic method for approximating a giving up smoking model containing fractional derivatives. Comput. Math. Appl. **64**(10), 3065–3074 (2012)
26. Zhang, Q., Luo, Y.H., Zhang, X.H., Wang, Y.: The relationship among school safety, school satisfaction, and students' cigarette smoking: based on a multilevel mediation model. Child Youth Serv. Rev. **88**, 96–102 (2018)
27. East, K., Hitchman, S.C., Bakolis, I., Williams, S., Cheeseman, H., Arnott, D., McNeill, A.: The association between smoking and electronic cigarette use in a cohort of young people 2017. J. Adolesc. Health **62**(5), 539–547 (2017)
28. Doran, N., Brikmanis, K., Petersen, A., Delucchi, K., Al-Delaimy, W.K., Luczak, S., Myers, M., Strong, D.: Does e-cigarette use predict cigarette escalation? A longitudinal study of young adult non-daily smokers. Prev. Med. **100**, 279–284 (2017)
29. Choi, W.S., Gilpin, E.A., Farkas, A.J., Pierce, J.P.: Determining the probability of future smoking among adolescents. Addiction **96**(2), 313–323 (2001)
30. Krugman, D.M.: Understanding the impact that marketing, advertising, and promotion have on adolescent E-cigarette behavior. J. Adolesc. Health **59**(6), 609 (2016)
31. Van den Driessche, P., Watmough, J.: Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission. Math. Biosci. **180**, 29–48 (2002). https://doi.org/10.1016/S0025-5564(02)00108-6
32. Dimitriu, G., Boiculese, V.L.: 2015 Sensitivity study for a SEIT epidemic model. In: 2015 E-Health and Bioengineering Conference (EHB), Iaşi, pp. 1–4 (2015). https://ieeexplore.ieee.org/abstract/document/7391495
33. Chitnis, N., Hyman, J.M., Cushing, J.M.: Determining important parameters in the spread of malaria through the sensitivity analysis of a mathematical model. Bull. Math. Biol. **70**, 1272–1296 (2008). https://doi.org/10.1007/s11538-008-9299-0
34. Pontryagin, L.S., Boltyanskii, V.G., Gamkrelidze, R.V., Mishchenko, E.F.: The Mathematical Theory of Optimal Processes. Wiley, New York (1962)
35. Pierce, J.P., Choi, W.S., Gilpin, E.A., Farkas, A.J., Merritt, R.K.: Validation of susceptibility as a predictor of which adolescents take up smoking in the United States. Health Psychol. **15**(5), 355 (1996)

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