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A flow model of corporate activities with quality assurance

Hong Zhang [a,b], Saviour Worlanyo Akuamoah [c], Paul Georgescu [d,*]

[a] School of Economics and Management, Changzhou Institute of Technology, Changzhou, JiangSu 213032, PR China
[b] Department of Financial Mathematics, Jiangsu University, ZhenJiang, JiangSu 212013, PR China
[c] Faculty of Science, Jiangsu University, ZhenJiang, JiangSu 212013, PR China
[d] Department of Mathematics, Technical University of Iaşi, Bd. Copou 11, 700506 Iaşi, Romania

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A B S T R A C T

We model a multi-divisional firm as a queueing system consisting in interconnected nodes, each project passing through queues, processing and quality assurance checks. It is assumed that the uptake of projects into processing is delayed by the availability of resources and by the size of the queue. The costs of delays are linearly dependent on queue lengths, assumption which is appropriate for short to medium stretches and instantaneous factors are assumed to account for the most significant costs, as opposed to long term discounting. Sufficient conditions for the existence, uniqueness and stability of the steady state, respectively, are determined via a matrix theory argument. We then find the optimal capacities, determining in the process an easily interpretable version of the optimality condition. Finally, the model is validated against real data from Damco Logistics Ghana Ltd. and it is seen that the profit would be maximized when the aggregate excess capacity is 71.8% of the aggregate optimal capacity during the data acquisition time.

1. Introduction

Corporate activities include all economic activities, often very diverse, carried out by a company during the course of its business [1,2]. Boeing, Airbus and other commercial aircrafts are made of as many as several million parts, which are produced by thousands of suppliers from dozens of countries around the world before being shipped and finally assembled at the facilities of the aircraft manufacturer, any delay in manufacturing being likely to have a significant financial impact on the parent company.

In today's competitive environment, joint corporate activities, including purchasing, are becoming a vital strategic issue for most small and medium-sized enterprises (SMEs). To gain leverage in negotiations for better pricing, services or technology, SMEs are motivated to join together or utilize an independent third party to assist them in their purchasing activities [3]. Consequently, joint corporate activities are becoming a part of competitive strategies across industries.

Companies and institutions benefits are typically driven by multiple objectives [4,5]. The complex requirements of resource allocation highlight the need for an approach that will enable decision makers to balance costs, risks and multiple benefits [6]. Resource allocation is taken to be the overall allocation of financial resources to decentralize management areas within an institution or a company, being closely related to budgeting, which is concerned with specific expenditure plans. When resources are scarce, queues are formed to cater for customers, hence the need of discussing queuing systems [7]. Queuing systems represent a particular example of a much broader class of interesting dynamic systems,

* Corresponding author.
E-mail addresses: zhanghong2018@czu.cn (H. Zhang), awalimemab@yahoo.com (S.W. Akuamoah), vpgeo@math.tuiasi.ro (P. Georgescu).

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called flow systems. A flow system is a system in which certain commodities are transferred through one or more finite-capacity channels in order to go from one point to another [8].

Prompt completion of corporate activities has become a prevalent business practice, a standard example being parcel delivery services such as FedEx, DHL and EMS that offer overnight delivery, ground shipping and airport delivery. Generally, when customers spend a long time at a place or institution for service, that institution is seen as delivering poor services and being in need to improve upon its performance [9], although queue forming can be seen as beneficial if queues can be managed so that both parties, namely those that wait in the queue and those that serve customers, derive maximal benefits [10]. A wide range of different techniques have been proposed to deal with queuing problems in institutions and companies. Kolmogorov's and Feller's works on purely discontinuous processes paved way for the foundation of Markov theory processes [11]. In [12], Rolland and Bazzoni investigated the behavior of queuing systems during a finite time interval, proposing queuing theories involving stochastic processes. Garg et al. presented in [13] a model of customers flow in a cost or capacity constrained institutions. A comprehensive multi-objective learning particle swarming optimization with a representation scheme based on binary search for products allocation problem was presented in Gong et al. [14]. In [15], Lee and Zenios developed a semi-closed migration network to optimize the flow of customers flowing into an institution. A queuing approach allowing for non-homogeneous arrival patterns, non-exponential service time distributions, and multiple patient types has been used in [16] along with a spreadsheet implementation of the resulting queuing equations to increase the capacity of an emergency department. In traditional models of queuing networks, such as those presented in Kleinrock [17] and Takagi [18], processors were fully occupied, though there might be delays in switching from task to task.

The main motivation of this work is the modeling of corporate activities in multi-divisional firms with multiple, interconnected processing nodes, as introduced in [19]. In this paper, we extend the value chain model proposed in [19] in order to allow products to undergo proper quality checking. This leads us to a higher dimensional problem and to certain changes in the coupling of the resulting ODE system, although conceptually the analysis in [19] is more comprehensive than the one done here, particularly with respect to the response to external changes. Also, our cost functional differs from the one employed in [19] in the fact that it also account for the costs associated with processing and quality assurance.

Our main objective is to model a queueing system, in which jobs or tasks within an organization flow through queues into processors then finally go through quality assurance checks before they are discharged to the next node. Additionally, the stability of the system is also discussed and optimal capacity conditions are determined. Further, we present an illustrative example describing the work of Damco Logistics Ghana Limited. This company was established in 1994 as a subsidiary of Maersk Ghana Limited, their services including customs clearance (Sea/Air/Land), inland transportation, warehousing, consolidation (To and From) worldwide, door to door deliveries (Sea and Air), freight forwarding (Sea and Air) and project and contracts management (Worldwide) [20].

The remaining part of this work is organized as follows. In Section 2, our model is presented, the steady state configuration of the network is examined and then stability results are given. Subsequently, we determine in Section 3 the unique set of optimal capacities which maximize the profit via an explicit formula. In Section 4, the above analysis is applied to a real-world problem of forecasting how changes in demand and supply prices affects optimal capacities and how quickly a Ghanaian firm should be prepared to react to such changes. Finally, concluding remarks are given in Section 5.

2. Mathematical modeling

2.1. Model description

We consider a queuing system consisting of N processing units, called nodes. Each node n $(n = 1, 2, \ldots, N)$ processes projects, an abstract denomination for the objects or tasks to be processed, and has projects in queue (q_n), projects in processing (p_n) and projects in quality assurance (s_n), an illustrative example being given in Fig. 1.

The variables of node n $(n = 1, 2, \ldots, N)$ in our model are defined in Table 1 below. Note that q_n, p_n, s_n are time-dependent, while c_n, a_n, b_n, h_n, d_n, ω_n, γ_n, δ_n, σ_n f_n, r_n, ρ_n are assumed to be constant. Although the latter assumption is, in itself, a simplification of reality, it is not a stretch to assume, for instance, that the marginal costs per project remain constant if the time span is short enough.

The transfer from queue to processing takes place at a rate that depends on the size of the queue q_n and the excess capacity for the queuing projects, defined as the difference between the capacity for projects in processing and the actual amount of projects in processing $c_n - p_n$ (see [19] for details). The rate of transfer flow from queue to processing is given by

$$\frac{q_n(c_n - p_n)}{a_n q_n + b_n(c_n - p_n)}.$$

The transfer from processing to quality assurance also depends on the size of the processing p_n. According to the result in [21], known as the Little's Theorem, $N = \lambda \mathcal{T}$, which expresses the natural idea that crowded systems (large N) are associated with long customer delays (large $\mathcal{T}$). For example, consider a network of transmission lines where packets

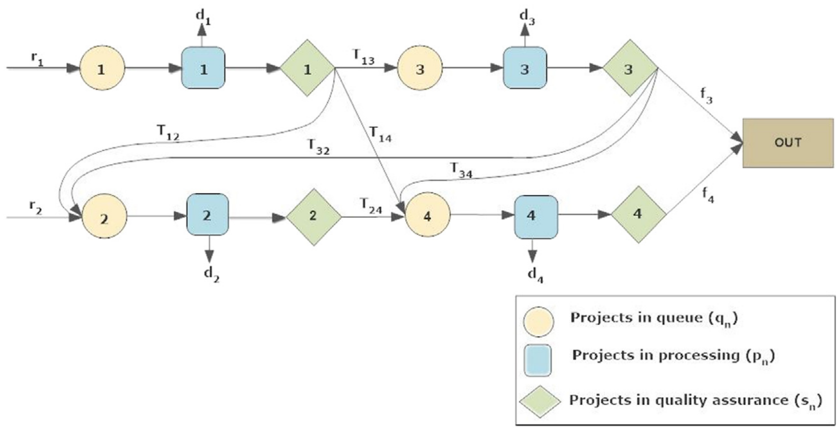

Fig. 1. Queuing network model ($N = 4$ for simplicity). From a logistics viewpoint, the processing nodes can be interpreted as raw material suppliers, factories, warehouses, distributors or retailers.

Table 1
Description of variables and coefficients used in node n.

Variable and coefficient	Description
q_n	Projects in queue.
p_n	Projects in processing.
s_n	Projects in quality assurance.
c_n	Capacity for projects in processing.
a_n	Accommodation delay for processing.
h_n	Delay for quality assurance.
b_n	Preprocessing delay for processing.
d_n	Projects that are removed (taken out).
f_n	Departures from quality check.
r_n	Inflow.
ρ_n	Price per project that emerges from the node.
ω_n	Marginal cost of capacity.
γ_n	Marginal cost per project in queue.
δ_n	Marginal cost per project in processing.
σ_n	Marginal cost per project in quality assurance.

arrive at n different nodes with corresponding rates $\lambda_1, \ldots, \lambda_n$. If N is the average total number of packets inside the network, then regardless of packet length distribution and method for routing packets, the average delay per packet is

$$\mathcal{T} = \frac{N}{\sum_n^{i=1} \lambda_i}.$$

Furthermore, the Little's Theorem also yields $N_i = \lambda_i \mathcal{T}_i$, where N_i and $\mathcal{T}_i$ are the average number of packets and the average delay of packets arriving at node i, respectively. The arrival rates of projects from processing to quality assurance are given by the average time $\frac{1}{h_n}$ spent by a project in processing and the average number p_n of projects waiting in processing [21]. Then by the Little's Theorem one obtains that the flow rate from processing to quality assurance is given by

$$\frac{p_n}{h_n}.$$

Some of the nodes accept projects from outside the system, which are interpreted as inflows, or taking assignments from customers and other external entities. We let r_n denote the rate at which nodes which can accept inflows of projects. Nodes can also send projects to other nodes. We denote T_{nm} as the rate at which projects in processing at node n spawn projects at node m. It is assumed that $T_{nn} = 0$ to help simplify some later expressions. Since a single project at node n can spawn multiple projects at several other nodes, it is possible that $\sum_m T_{nm} \geq 1$. Each node may also terminate or discharge a certain fraction of its transfer projects, causing them to exit the system. Let d_n denote the fraction of projects that are taken out from the system before processing commences and let f_n denote the fraction of projects in quality assurance at node n that are discharged per unit of time.

The description of the queue, processing and quality assurance of node n leads to the following ODE model

$$\dot{q}_n = r_n + \sum_{m=1}^{N} T_{mn} s_m - \frac{q_n(c_n - p_n)}{a_n q_n + b_n(c_n - p_n)} \doteq l_n, \tag{1}$$

$$\dot{p}_n = -d_n p_n + \frac{q_n(c_n - p_n)}{a_n q_n + b_n(c_n - p_n)} - \frac{p_n}{h_n} \doteq \kappa_n, \tag{2}$$

$$\dot{s}_n = -f_n s_n - s_n \sum_{m=1}^{N} T_{nm} + \frac{p_n}{h_n} \doteq \rho_n. \tag{3}$$

As observed in Eq. (1), projects from outside the system arrive into the queue of the nth node at a rate r_n, the negative term in Eq. (1) representing the part that is transferred to processing. The term d_n is the fraction of p_n that is removed from processing, as indicated in Eq. (2), the flow into p_n consisting exclusively in projects in the queue of node n, represented by the positive term in Eq. (2). The second negative term in Eq. (2) represents the transfer rate from processing to quality assurance. In Eq. (3), the flow out s_n, again by assumption, is proportional to s_n per unit of time, the fraction T_{nm} flows to the queue of node m and the positive term in Eq. (3) consists of projects in the queue at node n.

2.2. Steady state analysis

To find the steady states, we set the derivatives in Eqs. (1)–(3) to be zero for each node, that is,

$$\dot{q}_n = \dot{p}_n = \dot{s}_n = 0.$$

Then the total rate of change in the number of projects at node n is given by

$$\dot{q}_n + \dot{p}_n + \dot{s}_n = r_n + \sum_{m=1}^{N} T_{mn} s_m - d_n p_n - f_n s_n - s_n \sum_{m=1}^{N} T_{nm} = 0. \tag{4}$$

We let $\mu_n = f_n + \sum_{m=1}^{N} T_{nm}$ and define

$$A_{mn} = \begin{cases} T_{mn} & \text{if } m \neq n, \\ T_{nn} - \mu_n & \text{if } m = n. \end{cases} \tag{5}$$

With these notations, Eq. (4) can be restated as

$$d_n p_n^* - \sum_{m=1}^{N} A_{mn} s_m^* = r_n, \tag{6}$$

(q_n^*, p_n^*, s_n^*) being a nonnegative solution for the steady state process. From Eq. (3), we see that

$$\mu_n s_n^* = \frac{p_n^*}{h_n} \implies s_n^* = \frac{p_n^*}{\mu_n h_n}. \tag{7}$$

Substituting (7) into (6) gives

$$d_n p_n^* - \sum_{m=1}^{N} A_{mn} \frac{p_m^*}{\mu_m h_m} = r_n, \quad n = 1, 2, \ldots, N,$$

that is,

$$\begin{pmatrix} d_1 & & & & \\ & d_2 & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & d_N \end{pmatrix} \begin{pmatrix} p_1^* \\ p_2^* \\ \cdot \\ \cdot \\ p_N^* \end{pmatrix} - \begin{pmatrix} \frac{A_{11}}{\mu_1 h_1} & \frac{A_{21}}{\mu_2 h_2} & \cdot & \cdot & \cdot & \frac{A_{N1}}{\mu_N h_N} \\ \frac{A_{12}}{\mu_1 h_1} & \frac{A_{22}}{\mu_2 h_2} & \cdot & \cdot & \cdot & \frac{A_{N2}}{\mu_N h_N} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \frac{A_{1N}}{\mu_1 h_1} & \frac{A_{2N}}{\mu_2 h_2} & \cdot & \cdot & \cdot & \frac{A_{NN}}{\mu_N h_N} \end{pmatrix} \begin{pmatrix} p_1^* \\ p_2^* \\ \cdot \\ \cdot \\ p_N^* \end{pmatrix} = \begin{pmatrix} r_1 \\ r_2 \\ \cdot \\ \cdot \\ r_N \end{pmatrix}.$$

Let us define

$$\lambda_{ij} = \begin{cases} d_i - \dfrac{A_{ji}}{\mu_j h_j}, i = j, \\ -\dfrac{A_{ji}}{\mu_j h_j}, i \neq j, \end{cases}$$

$$p^* = \begin{pmatrix} p_1^* \\ p_2^* \\ \cdot \\ \cdot \\ p_N^* \end{pmatrix}, \quad M = \begin{pmatrix} \lambda_{11} & \lambda_{12} & \cdot & \cdot & \cdot & \lambda_{1N} \\ \lambda_{21} & \lambda_{22} & \cdot & \cdot & \cdot & \lambda_{2N} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \lambda_{N1} & \lambda_{N2} & \cdot & \cdot & \cdot & \lambda_{NN} \end{pmatrix} \quad \text{and} \quad r = \begin{pmatrix} r_1 \\ r_2 \\ \cdot \\ \cdot \\ r_N \end{pmatrix}.$$

Let us assume that

$$d_n > \sum_{m=1}^{N} \frac{A_{mn}}{\mu_m h_m}, \quad n = 1, 2, \ldots, N. \tag{8}$$

This condition establishes the dominance of the diagonal in the matrix M. Consequently, M is nonsingular. Since M is a Z-matrix, i.e., a matrix all of whose off-diagonal entries are non-positive, it follows that its inverse has nonnegative entries [19]. Since the inflows r_n are nonnegative, the steady state process sizes, given by

$$p^* = (M)^{-1} r, \tag{9}$$

are also nonnegative.

It is to be noted that, although sufficient only and perhaps without a transparent concrete interpretation, condition (8) is, in a sense, natural and appropriate, as it ensures that the steady state process sizes not only exists and are unique, but are also nonnegative. If no such condition is assumed, the steady states may not exist, may not be unique, or may have negative components.

Let us also denote

$$s^* = \begin{pmatrix} s_1^* \\ s_2^* \\ \cdot \\ \cdot \\ \cdot \\ s_N^* \end{pmatrix}, \quad \Gamma = diag\left(\frac{1}{\mu_i h_i}\right)_{1 \le i \le n}.$$

From Eq. (7), the steady state quality assurance solution is given by

$$s^* = \Gamma p^* = \Gamma M^{-1} r. \tag{10}$$

The steady state queue sizes $q_1^*, q_2^*, \ldots, q_n^*$ can be determined from Eq. (1), which leads to

$$r_n + \sum_{m=1}^{N} T_{mn} s_m^* - \frac{q_n^*(c_n - p_n^*)}{a_n q_n^* + b_n(c_n - p_n^*)} = 0.$$

We then denote $T_{.n} = (T_{1n}, T_{2n}, \ldots, T_{Nn})$ as the transfer probability of projects in quality assurance at different nodes to the queue of node n and let

$$\tilde{r}_n = T_{.n} \Gamma M^{-1} r,$$

which implies that

$$r_n + \tilde{r}_n - \frac{q_n^*(c_n - p_n^*)}{a_n q_n^* + b_n(c_n - p_n^*)} = 0.$$

We then denote $\beta_n = r_n + \tilde{r}_n$, which gives

$$\beta_n = \frac{q_n^*(c_n - p_n^*)}{a_n q_n^* + b_n(c_n - p_n^*)},$$
$$q_n^* = \frac{\beta_n b_n (c_n - p_n^*)}{c_n - (p_n^* + \beta_n a_n)}. \tag{11}$$

Let us denote

$$c = (c_1, c_2, \ldots, c_N)^T, \quad \beta = (\beta_1 a_1, \beta_2 a_2, \ldots \beta_N a_N)^T.$$

We then derive the following result.

Theorem 2.1. *The condition*

$$c > M^{-1} r + \beta \tag{12}$$

(understood component-wise), along with (8), guarantee the existence and uniqueness of the positive steady state of Eqs. (1)–(3).

To prove that the positive steady state is locally stable, we linearize the system by finding the Jacobian matrix of $(\dot{q}_1, \ldots, \dot{q}_n, \dot{p}_1, \ldots, \dot{p}_n, \dot{s}_1, \ldots, \dot{s}_n)^T$ at the steady state configuration.

Defining

$$J(\dot{q_n}, \dot{p_n}, \dot{s_n}) = \begin{pmatrix} \frac{\partial l_n}{\partial q_n} & \frac{\partial l_n}{\partial p_n} & \frac{\partial l_n}{\partial s_n} \\ \frac{\partial \kappa_n}{\partial q_n} & \frac{\partial \kappa_n}{\partial p_n} & \frac{\partial \kappa_n}{\partial s_n} \\ \frac{\partial \rho_n}{\partial q_n} & \frac{\partial \rho_n}{\partial p_n} & \frac{\partial \rho_n}{\partial s_n} \end{pmatrix},$$

in which

$$
\begin{cases}
\dfrac{\partial l_n}{\partial q_n} = -\, b_n \left[\dfrac{(c_n - p_n^*)}{a_n q_n^* + b_n(c_n - p_n^*)} \right]^2 \doteq -R_n, \\
\dfrac{\partial l_n}{\partial p_n} = a_n \left[\dfrac{q_n^*}{a_n q_n^* + b_n(c_n - p_n^*)} \right]^2 \doteq Q_n, \\
\dfrac{\partial l_n}{\partial s_1} = T_{1n},\ \dfrac{\partial l_n}{\partial s_2} = T_{2n},\ \dots,\ \dfrac{\partial l_n}{\partial s_N} = T_{Nn}, \\
\dfrac{\partial \kappa_n}{\partial q_n} = b_n \left[\dfrac{(c_n - p_n^*)}{a_n q_n^* + b_n(c_n - p_n^*)} \right]^2 \doteq R_n, \\
\dfrac{\partial \kappa_n}{\partial p_n} = -\, d_n - a_n \left[\dfrac{q_n^*}{a_n q_n^* + b_n(c_n - p_n^*)} \right]^2 - \dfrac{1}{h_n} \doteq -U_n, \\
\dfrac{\partial \kappa_n}{\partial s_n} = \dfrac{\partial \rho_n}{\partial q_n} = 0, \\
\dfrac{\partial \rho_n}{\partial p_n} = \dfrac{1}{h_n}, \\
\dfrac{\partial \rho_n}{\partial s_n} = -\,(f_n + \sum_{m=1}^{N} T_{nm}) \doteq -\mu_n,
\end{cases}
$$

the Jacobian matrix is then given by

$$
J = \left(\begin{array}{cccc|cccc|cccc}
-R_1 & & & & Q_1 & & & & T_{11} & T_{21} & \cdots & T_{N1} \\
 & -R_2 & & & & Q_2 & & & T_{12} & T_{22} & \cdots & T_{N2} \\
 & & \ddots & & & & \ddots & & \vdots & \vdots & \ddots & \vdots \\
 & & & -R_N & & & & Q_N & T_{1N} & T_{2N} & \cdots & T_{NN} \\
R_1 & & & & -U_1 & & & & 0 & & & \\
 & R_2 & & & & -U_2 & & & & 0 & & \\
 & & \ddots & & & & \ddots & & & & \ddots & \\
 & & & R_N & & & & -U_N & & & & 0 \\
0 & & & & \frac{1}{h_1} & & & & -\mu_1 & & & \\
 & 0 & & & & \frac{1}{h_2} & & & & -\mu_2 & & \\
 & & \ddots & & & & \ddots & & & & \ddots & \\
 & & & 0 & & & & \frac{1}{h_N} & & & & -\mu_N
\end{array} \right), \tag{13}
$$

the empty entries being filled with zeros. We will further show that the matrix has a strictly dominant diagonal.

Theorem 2.2. *If the condition (12) and the following inequalities hold*

$$
\begin{cases}
d_n + \dfrac{1}{h_n} > R_n - Q_n > \displaystyle\sum_{m=1}^{N} T_{mn}, \\
h_n \mu_n > 1,
\end{cases} \tag{14}
$$

then the positive steady state of the system (1)–(3) is locally stable

Proof. Note first that condition (14) implies that the inequality (8) holds. Also, condition (14) gives the following three inequalities

(1) $R_n > Q_n + T_{1n} + T_{2n} + \cdots + T_{Nn},\ n = 1, 2, 3, \dots, N.$
(2) $U_n > R_n,\ n = 1, 2, \dots, N.$
(3) $\mu_n > \frac{1}{h_n},\ n = 1, 2, \dots, N.$

It then follows that the matrix J in (13) is strictly diagonally dominant. According to McKenzie [18] (Theorem 2), J is negative definite, implying that the system is locally stable around the steady state.

As per its definition, a locally stable equilibrium can withstand, in a certain sense, only arbitrarily small perturbations. For practical purposes, it would be preferable to estimate the size of the basin of attraction or to complement the local analysis by considering several nonlocal stability and resilience measures built upon numerical estimates of the basin and on the return time of trajectories corresponding to perturbations from the equilibrium in order to characterize the ability of the equilibrium to withstand larger perturbations as well [22].

3. Optimal capacities

3.1. Completion time

The number of projects in quality control does not depend on capacity in the steady state, as seen from Eqs. (9) and (10), but the queue length and time to completion do. We let t_n denote the average total amount of time needed to process a project at node n and then sent to the next node, and define t as the column vector of all these values. For steady states, $\dot{q}_n = \dot{p}_n = \dot{s}_n = 0$ and hence the outflow of queue n is

$$\mu_n s_n^* + d_n p_n^* = r_n + \sum_{m=1}^{N} T_{nm} s_m^*.$$

Then from Eq. (11), the total time a project stays in queue n is the size of the queue divided by the outflow, that is

$$\frac{q_n^*}{\mu_n s_n^* + d_n p_n^*} = \frac{\beta_n b_n (c_n - p_n^*)}{\big(\mu_n s_n^* + d_n p_n^*\big)\big(c_n - (p_n^* + \beta_n a_n)\big)}.$$

The outflow from processing at node n is $d_n p_n^*$. From Eq. (9), the amount of time a project spends in processing is given by

$$\frac{p_n^*}{d_n p_n^*} = \frac{1}{d_n}.$$

The outflow of quality assurance is $\mu_n s_n^*$. From Eq. (10) the amount of time a project spends in quality assurance is also given by

$$\frac{s_n^*}{\mu_n s_n^*} = \frac{1}{\mu_n}.$$

Thus, the time spent at node n is

$$\begin{aligned}\tau_n &= \frac{\beta_n b_n (c_n - p_n^*)}{(\mu_n s_n^* + d_n p_n^*)\big(c_n - (p_n^* + \beta_n a_n)\big)} + \frac{1}{d_n} + \frac{1}{\mu_n},\\ &= \frac{1}{(\mu_n s_n^* + d_n p_n^*)}\left(\frac{(d_n + \mu_n)(d_n p_n^* + \mu_n s_n^*)\big(c_n - (p_n^* + \beta_n a_n)\big) - \big(b_n \beta_n d_n \mu_n (c_n - p_n^*)\big)}{d_n \mu_n \big(c_n - (p_n^* + \beta_n a_n)\big)}\right).\end{aligned}$$

We then let $\phi_n = \mu_n s_n^* + d_n p_n^*$ and define a column vector z so that, for each n,

$$z_n = \frac{(d_n + \mu_n)(d_n p_n^* + \mu_n s_n^*)\big(c_n - (p_n^* + a_n \beta_n)\big) - \big(b_n \beta_n d_n \mu_n (c_n - p_n^*)\big)}{d_n \mu_n \big(c_n - (p_n^* + a_n \beta_n)\big)},$$

where

$$\big[(d_n + \mu_n)(d_n p_n^* + \mu_n s_n^*)\big(c_n - (p_n^* + \beta_n a_n)\big)\big] + b_n \beta_n d_n \mu_n p_n^* > b_n \beta_n d_n \mu_n c_n,$$

(since $c > M^{-1} r + \beta$, the numerator of the column vectors are nonnegative and well defined).

The processing time of a project at a latter node does not depend on the amount of time spent in a previous node. To ground our model in reality, we assume that the total outflow per unit at any node does not exceed the number of units in processing, that is, $\mu_n s_n^* + d_n p_n^* \leq p_n^*$. The average total time in connection with the projects sent to node m is denoted by t_m, which is multiplied by the probability that a random project is sent there conditionally on existing $\frac{T_{nm}}{\mu_n s_n^* + d_n p_n^*}$, giving a recursive equation for the amount of time required to process a project.

$$t_n = \frac{z_n}{\phi_n} + \sum_{m=1}^{N} \frac{T_{nm}}{\phi_n} t_m \implies \phi_n t_n - \sum_{m=1}^{N} T_{nm} t_m = z_n.$$

In matrix form, this can be written as

$$\begin{pmatrix} \phi_1 & & & & & \\ & \phi_1 & & & & \\ & & \cdot & & & \\ & & & \cdot & & \\ & & & & \cdot & \\ & & & & & \phi_N \end{pmatrix} \begin{pmatrix} t_1 \\ t_2 \\ \cdot \\ \cdot \\ \cdot \\ t_N \end{pmatrix} - \begin{pmatrix} T_{11} & T_{12} & \cdot & \cdot & \cdot & T_{1N} \\ T_{21} & T_{22} & \cdot & \cdot & \cdot & T_{2N} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ T_{N1} & T_{N2} & \cdot & \cdot & \cdot & T_{NN} \end{pmatrix} \begin{pmatrix} t_1 \\ t_2 \\ \cdot \\ \cdot \\ \cdot \\ t_N \end{pmatrix} = \begin{pmatrix} z_1 \\ z_2 \\ \cdot \\ \cdot \\ \cdot \\ z_N \end{pmatrix},$$

that is

$$(H - T)t = z,$$

in which

$$H = \begin{pmatrix} \phi_1 & & & & & \\ & \phi_2 & & & & \\ & & \cdot & & & \\ & & & \cdot & & \\ & & & & \cdot & \\ & & & & & \phi_N \end{pmatrix}, \quad t = \begin{pmatrix} t_1 \\ t_2 \\ \cdot \\ \cdot \\ \cdot \\ t_N \end{pmatrix}, \quad T = \begin{pmatrix} T_{11} & T_{12} & \cdot & \cdot & \cdot & T_{1N} \\ T_{21} & T_{22} & \cdot & \cdot & \cdot & T_{2N} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ T_{N1} & T_{N2} & \cdot & \cdot & \cdot & T_{NN} \end{pmatrix} \text{ and } z = \begin{pmatrix} z_1 \\ z_2 \\ \cdot \\ \cdot \\ \cdot \\ z_N \end{pmatrix}.$$

Then

$$\phi_n > \sum_{m=1}^{N} T_{nm},$$

implies the dominance of the diagonal. The completion times for the model has strictly nonnegative entries and is well-defined. Thus

$$t = (H - T)^{-1} z. \tag{15}$$

3.2. Optimal problem

The aim of a company is to boost the average profit acquired per unit of time. In our settings, the revenues dwell on payments for the completion of projects that are authorized at each node. In this regard, we let ρ_k denote the price paid to the company for projects that emerge from node k and categorize costs into five types: costs of capacity, queue, processor, delays in project to completion and cost of projects that are rejected at quality assurance.

We assume that capacity at node k has a constant marginal cost $\omega_k > 0$ in units of money per project per unit of time, the queue has a constant marginal cost γ_k in units of money per project per unit of time, the constant marginal cost of the processor is denoted by δ_k and σ_k denotes the constant marginal cost of projects not meeting requirements at quality control per project per unit of time. Assuming that a project originating at node k takes t_k units of time to complete, the company pays a time premium $f_k(t_k)$, in units of money per project. The profit function for the company is then given by

$$\pi = \sum_{k=1}^{N} \left[\rho_k r_k - \omega_k c_k - \gamma_k q_k^* - \delta_k p_k^* - \sigma_k s_k^* - r_k \mathbb{E}(f_k(t_k)) \right]. \tag{16}$$

Assuming that the inflows and transfer rates are fixed, the only endogenous variables in the profit function are capacity, queue size, the processor, quality assurance and time to completion. Maximizing the profit, we choose the vector of capacities c to minimize the function given by

$$K(c) = \sum_{k=1}^{N} \left[\omega_k c_k + \gamma_k q_k^* + \delta_k p_k^* + \sigma_k s_k^* + r_k \mathbb{E}(f_k(t_k)) \right]. \tag{17}$$

Modeling the costs of delays as linear functions, we have $f_k(t_k) = \xi_k t_k$, for a constant ξ_k, which we call the urgency of projects originating at node k. Given any distribution on completion times with mean t_k, we have

$$\mathbb{E}(f_k(t_k)) = \xi_k t_k.$$

Let us denote

$$(G_{mk})_{1 \le m \le N,\, 1 \le k \le N} = (H - T)^{-1}.$$

Eq. (17) then gives

$$K(c) = \sum_{k=1}^{N} \left[\omega_k c_k + \gamma_k \left(\frac{\beta_k b_k (c_k - p_k^*)}{c_k - (p_k^* + \beta_k a_k)} \right) + \delta_k p^* + \sigma_k s_k^* + \xi_k r_k \sum_{m=1}^{N} G_{mk} \right.$$

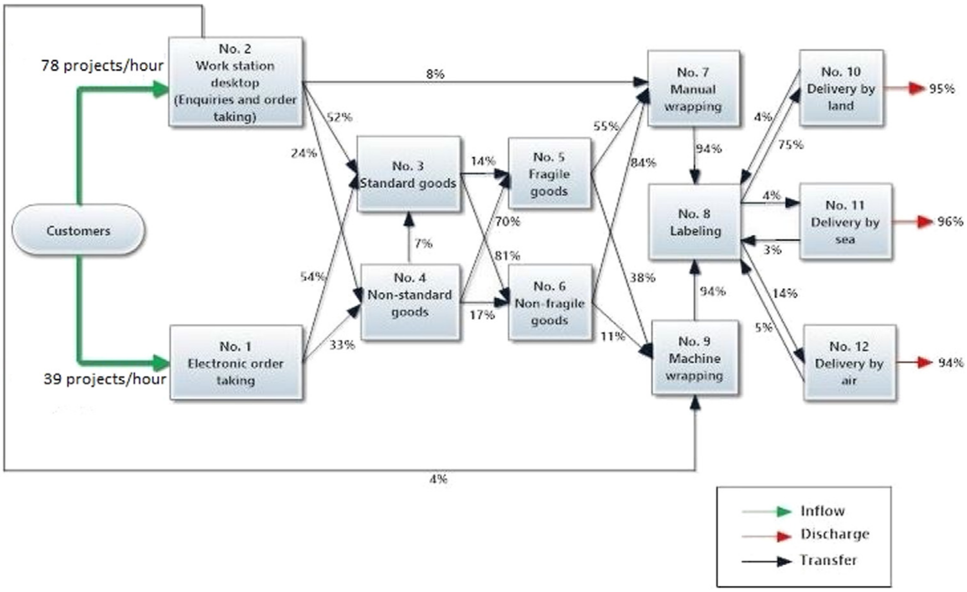

Fig. 2. Logistic model of corporate activities.

$$\times\left(\frac{(d_k+\mu_k)(d_kp_k^*+\mu_ks_k^*)\big(c_k-(p_k^*+a_k\beta_k)\big)}{d_k\mu_k\big(c_k-(p_k^*+a_k\beta_k)\big)} - \frac{\big(b_k\beta_kd_k\mu_k(c_k-p_k^*)\big)}{d_k\mu_k\big(c_k-(p_k^*+a_k\beta_k)\big)}\right)\Bigg]. \tag{18}$$

The optimal capacities $c^* = (c_1^*, \ldots, c_N^*)^T$ which minimize K, if existing, must satisfy for each n the first order condition

$$\frac{\partial K}{\partial c_n} = \omega_n - \frac{\gamma_n a_n b_n \beta_n^2}{\big(c_n-(p_n^*+\beta_n a_n)\big)^2} - \frac{a_n b_n \beta_n^2}{\big(c_n-(p_n^*+\beta_n a_n)\big)^2}\xi_n r_n \sum_{m=1}^{N} G_{nm} = 0.$$

The equation above reflects the fact that p_n^* and s_n^* are determined independently of capacity. Solving the above equation, we obtain

$$c_n^* = p_n^* + \beta_n a_n + \beta_n\sqrt{\frac{a_n b_n}{\omega_n}\Big(\gamma_n + \xi_n r_n\sum_{k=1}^{N} G_{nk}\Big)},\ \ n = 1, 2, \ldots, N. \tag{19}$$

Here, c_n^* is well defined because the expression under the square root is nonnegative. We find that the capacities are optimal exactly when, for each node, we have

$$\begin{aligned}\omega_n &= \frac{\beta_n^2 a_n b_n}{(c_n^*-(p_n^*+\beta_n a_n))^2}\Big(\gamma_n + \xi_n r_n\sum_{m=1}^{N} G_{nm}\Big) \\ &= \frac{a_n}{b_n}\left(\frac{q_n^*}{c_n^*-p_n^*}\right)^2\Big(\gamma_n + \xi_n r_n\sum_{m=1}^{N} G_{nm}\Big),\ \ n = 1, 2, \ldots, N.\end{aligned} \tag{20}$$

In the right-hand side of the second line, the first factor represents the ratio of the accommodation delay to the preprocessing delay, the second factor is the ratio of the queue size to the excess capacity, squared, while the third factor is the marginal cost of the queue at the nth node plus the total cost of the elasticity of the urgencies in cooperation. That is, the optimal capacities are such that at each node n, the marginal cost of capacity equals the product on the right hand side (see [19] for details).

4. Numerical simulations

The empirical test for our model in Fig. 2 is based on the collected raw data from Damco Logistics Ghana Co., Ltd. from 9am to 5pm on 7th October, 2017. The illustrative example represents a delivery company offering a full range of integrated delivery services to customers across the globe.

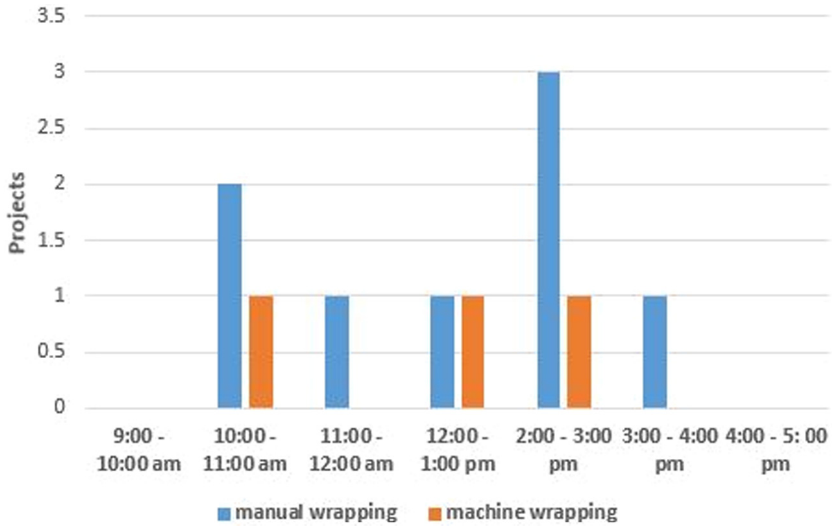

Fig. 3. Projects from node 2 to node 7 and, respectively, node 9 in the queuing network.

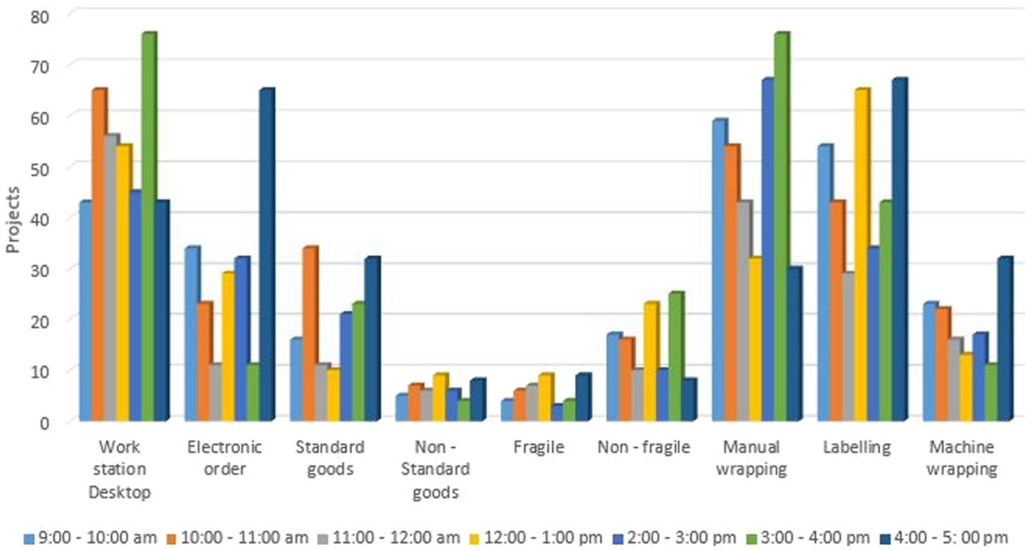

Fig. 4. Projects in nodes 1–9 of the queuing network.

The company has two receiving nodes, Electronic order taking, which receives new projects that are not in stock and Work station desktop order taking, which receives orders of new projects that are in stock. In Fig. 2, 234 projects was ordered electronically, with an average of 39 projects per hour and at Work station desktop, 468 projects were also ordered with an average of 78 per hour within the given time interval.

Of all projects coming through Work station and Electronic receiving nodes 12% and 11%, respectively, are mistakes and are taken out. Of all Electronic order projects, 33% are routed to the Non-standard goods node and 54% are routed to the Standard goods node. Similarly, of all projects coming through the Work station desktop, 52% are sent to Standard goods node and 24% are sent to Non-standard goods node for more extensive services. Also, 8% of the projects are routed to Manual wrapping node and 4% are sent to the Machine wrapping node. Additionally, 7% of the Non-standard goods are sent to Standard goods node, 17% sent to Non-fragile goods node and 70% to Fragile goods node. Similarly, 14% of the Standard goods are sent to Fragile goods node and 81% are sent to Non-fragile goods node. After quality check, 55% of the Fragile goods projects are sent to the Manual wrapping node and 38% are sent to Machine wrapping node. Likewise, 84% of Non-fragile goods are sent to the Manual wrapping node and 11% are sent to the Machine wrapping node. After quality check at Manual and Machine wrapping nodes, projects are routed to Labeling node. 95% of the projects sent to Labeling node are discharged by road, 96% are discharged by sea and 94% are also discharged by air.

Table 2
The accommodation and preprocessing delays (hour).

Stage	Accommodation delay	Preprocessing delay
Electronic order	0.03	0.06
Workstation desktop	0.05	0.06
Standard goods	0.08	0.10
Non-standard goods	0.08	0.10
Fragile goods	0.10	0.33
Non-Fragile goods	0.08	0.33
Manual wrapping	0.10	0.33
Labeling	0.10	0.33
Machine wrapping	0.08	0.33
Delivery by land	0.75	0.83
Delivery by sea	0.75	0.83
Delivery by air	0.75	0.83

Table 3
The capacity and queueing costs ($ per hour).

Stage	Capacity cost	Queueing cost
Electronic order	8	500
Workstation desktop	16	500
Standard goods	80	0.8
Non-standard goods	80	0.8
Fragile goods	80	0.8
Non-Fragile goods	80	0.8
Manual wrapping	80	0.8
Labeling	80	0.8
Machine wrapping	80	0.8
Delivery by land	80	0.8
Delivery by sea	80	0.8
Delivery by air	80	0.8

For technical reasons, all nodes that are not openly shown discharging projects discharge 1% of their projects per unit time (which corresponds to projects being canceled at some intermediate point with small probability).

According to Fig. 2, customers can send goods directly from Work station desktop to Manual wrapping and Machine wrapping nodes. Fig. 3 shows that 11 customers' goods to different clients within the time interval 9 : 00 am to 5 : 00 pm with highest of 3 customers' goods from Work station to Manual wrapping node within the time interval 2:00–3:00 pm. In Fig. 3, zero customer service interactions were recorded in the time intervals 9:00–10:00 am and 4:00–5:00 pm, respectively.

Fig. 4 depicts the fraction of remaining projects left for processing. In this regard, comparatively fewer projects were received for electronic processing.

The parameter a_n is the time it takes a server at node n to check for a new order. From Table 2, this time is short for electronic and workstation desktop order taking (under 3 min) and 10 min at Fragile, Manual wrapping and Labeling node. It is longer, however, at the Delivery node, where workers check for new assignments about every 75 min when idle. The parameter b_n corresponds to preprocessing delays and it is set as shown in Table 2. Again, this time is longer at the Delivery node. Capacity and queueing costs are shown in Table 3. The cost of employing one server (unskilled worker) is $1 per hour project, where as a skilled worker receives $10 per hour. We assume as a fact that keeping customers waiting in queues at Electronic and Work station order taking for a fraction x of an hour will result in 500x$ lost future profits for the company. Corresponding to the cost of goods at other nodes, queueing costs are quite low since a customer may not mind if his project takes 5 days to finish and ship, but may not be happy for being put on hold on the telephone for 40 min. Urgencies at the receiving nodes are 4 and 2 projects.

The transfer matrix is

$$T = \begin{bmatrix} 0 & 0 & 0.54 & 0.33 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.52 & 0.24 & 0 & 0 & 0.08 & 0 & 0.04 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.14 & 0.81 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.07 & 0 & 0.7 & 0.17 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.55 & 0 & 0.38 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.84 & 0 & 0.11 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.94 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.75 & 0.04 & 0.14 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.94 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.04 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.04 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.05 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Table 4
The steady process sizes, optimal capacities, steady queue sizes and steady quality assurances in the example.

Stage	Process sizes	Optimal capacity	Queue sizes Queue sizes	Quality assurance
Electronic order	715	835	212	635
Workstation desktop	615	705	211	445
Standard goods	2205	2443	213	1505
Non-standard goods	1300	1343	230	1255
Fragile goods	2706	2742	329	2505
Non-Fragile goods	5306	5555	344	5130
Manual wrapping	20800	21321	8738	20235
Labeling	26600	261390	384224	26510
Machine wrapping	5905	6318	3555	5600
Delivery by land	25600	25766	665	25505
Delivery by sea	525	679	266	510
Delivery by air	769	963	297	747

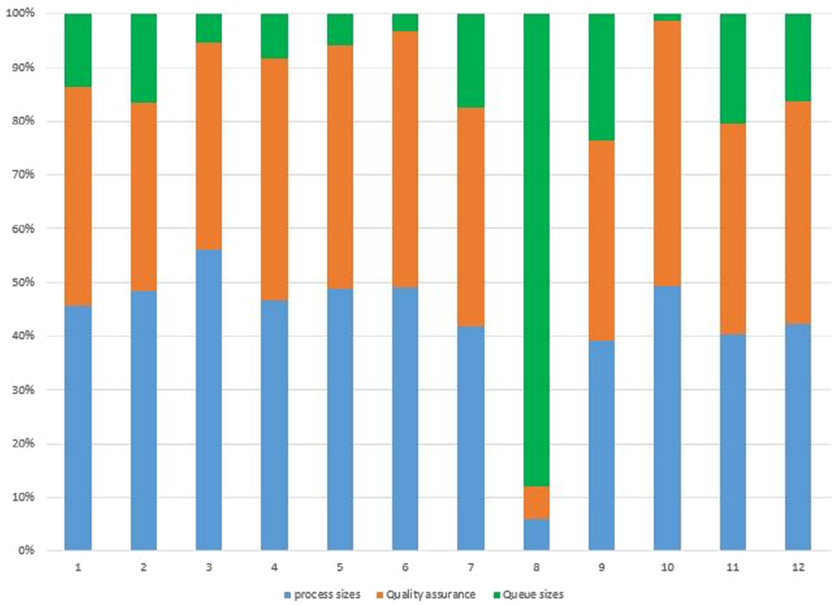

Fig. 5. An illustration of the steady process sizes, optimal capacities, steady queue sizes and steady quality assurances.

The inflows of projects into the company within the given time interval are $r_1 = 78$ projects per hour and $r_2 = 39$ projects per hour, with the rest of the entries being set to zero.

The fraction of projects that are taken out from the system before processing commences are

$$d = \text{diag}[0.12, 0.11, 0.04, 0.05, 0.06, 0.04, 0.05, 0.06, 0.05, 0.01, 0.01, 0.01].$$

The discharge rates are

$$f = \text{diag}[0.01, 0.01, 0.01, 0.01, 0.01, 0.01, 0.01, 0.01, 0.01, 0.95, 0.96, 0.94].$$

The delay of projects from processing to quality assurance is given by

$$h = \text{diag}[0.24, 0.36, 0.24, 0.34, 0.33, 0.32, 0.68, 0.41, 0.71, 0.014, 0.093, 0.011].$$

The steady state process sizes, queue sizes, quality assurances check and capacities are recorded in Table 4 and plotted in Fig. 5. According to Table 4 and Fig. 5, capacities are significantly larger than the process sizes at the same nodes, the reason being that idle processors do not check for new projects instantly. To avoid congestion or blockage, there must be enough available processors at any given time to ensure a sufficiently high rate of uptake from the queue.

It is evident from Table 5 that profit is maximized when the aggregate excess capacity, defined as the sum of all excess capacities for each node, is 71.8% of the aggregate optimal capacity, defined as the sum of all optimal capacities for each node, during the day. Note that, in this situation, the excess capacity at a given node represents the difference between the optimal capacity and the steady amount of processes at this node.

Table 5
Excess capacity.

Stage	Number of unused capacity
Electronic order	120
Workstation desktop	90
Standard goods	238
Non-standard goods	43
Fragile goods	36
Non-Fragile goods	249
Manual wrapping	521
Labeling	234790
Machine wrapping	413
Delivery by land	166
Delivery by sea	154
Delivery by air	194
Total	237014

Table 6
Numerical simulation for node 8 and node 10.

Stage	Project in queue	Project in processing	Project in quality control
Labeling	8738	26600	20235
Delivery by land	665	25600	25505

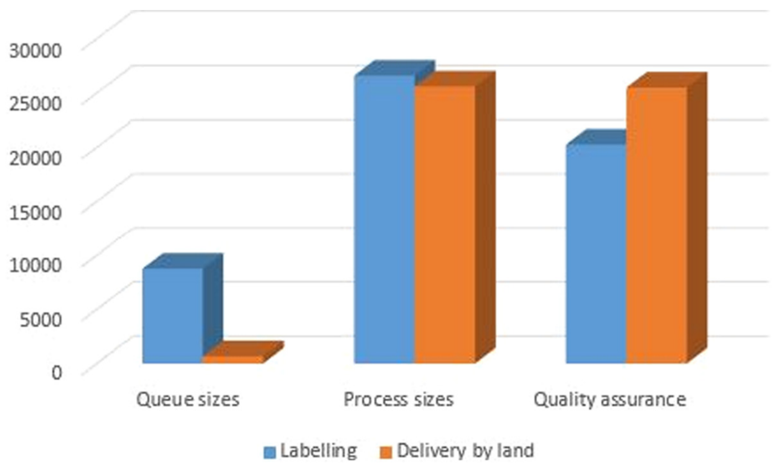

Fig. 6. An illustration of a few extracts from Fig. 2.

From Fig. 2, we extract Table 6 for node 8 and node 10 to plot Fig. 6 and verify that conditions in Theorems 2.1 and 2.2 are met. It is seen that

$$
\begin{aligned}
c &= \begin{pmatrix} 835 & 705 & 2443 & 1343 & 2742 & 5255 & 21321 & 261390 & 6318 & 25766 & 679 & 963 \end{pmatrix}^T \\
&> \\
M^{-1}r+\beta &= \begin{pmatrix} 736 & 638 & 2226 & 1321 & 2727 & 5327 & 20821 & 26621 & 5926 & 25621 & 546 & 790 \end{pmatrix}^T.
\end{aligned}
$$

Also, we have

$$
\begin{aligned}
d_n+\frac{1}{h_n} &= \begin{pmatrix} 4.18 & 2.79 & 4.17 & 2.95 & 3.13 & 3.135 & 14.72 & 2.44 & 3.23 & 25.34 & 15.23 & 19.63 \end{pmatrix}^T \\
&> \\
R_n-Q_n &= \begin{pmatrix} 2.02 & 1.02 & 3.001 & 2.002 & 2.03 & 1.59 & 10.11 & 1.09 & 15.03 & 11.09 & 546 & 10.11 \end{pmatrix}^T \\
&> \\
\sum_{m=1}^{N} T_{mn} &= \begin{pmatrix} 0.87 & 0.88 & 0.95 & 0.94 & 0.93 & 0.95 & 0.94 & 0.93 & 0.94 & 0.04 & 0.04 & 10.05 \end{pmatrix}^T.
\end{aligned}
$$

5. Conclusion

In this paper, we have introduced a model for the flow of corporate activities with projects in queue, projects in processing and projects in quality assurance. This model has been analyzed from the viewpoint of finding conditions for optimal resource allocation. Based on a matrix theory argument, we obtained a sufficient condition to guarantee the existence and uniqueness of the steady state. We then found a sufficient conditions of the local stability of the system, again via a matrix theory argument. Using linear cost assumptions, we also obtained the optimal capacities for the model, expressed using an elasticity condition, that is, a relation involving price and demand, providing a precise calculation of the effect of a change in price on quantity demanded. For real data from Damco Logistics Ghana Ltd, it has been determined that profit would be maximized when the aggregate excess capacity is 71.8% of the aggregate optimal capacity during the given time interval.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

CRediT authorship contribution statement

Hong Zhang: Conceptualization, Software, Formal analysis, Writing - original draft, Writing - review & editing. **Saviour Worlanyo Akuamoah:** Conceptualization, Software, Formal analysis, Writing - original draft. **Paul Georgescu:** Formal analysis, Writing - original draft, Writing review & editing.

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