Probleme propuse:

1. Calculaţi:

a) $\lim_{n\to\infty}\left(4-3n^4\right)$; b) $\lim_{n\to\infty}\left(3n^4-4\right)$; c) $\lim_{n\to\infty}\left(3n^6-6n^3\right)$; d) $\lim_{n\to\infty}\left(6n^3-3n^6\right)$; e) $\lim_{n\to\infty}\left(7n^5-5n^7\right)$; f) $\lim_{n\to\infty}\left(5n^7-7n^5\right)$ g) $\lim_{n\to\infty}\left(8n^4-4n^8\right)$; h) $\lim_{n\to\infty}\left(4n^8-8n^4\right)$.

2. Calculaţi:

a) $\lim_{n\to\infty}\frac{9n^3-2n^4+8n-1}{3n^3-5n^2+3}$; b) $\lim_{n\to\infty}\frac{9n^3-2n^4+8n-1}{3n^2-5n^3+3}$; c) $\lim_{n\to\infty}\frac{5n^3+3n^4-4n^2-3}{4n^4-3n^5+1}$; d) $\lim_{n\to\infty}\frac{5n^4+3n^3-4n^2-3}{4n^4-3n^5+1}$; e) $\lim_{n\to\infty}\frac{4n^5-3n^6+8n^4-1}{3n^5-5n^4+3}$; f) $\lim_{n\to\infty}\frac{4n^5-3n^6+8n^4-1}{3n^4-5n^5+3}$; g) $\lim_{n\to\infty}\frac{2n^4-4n^6+3n^5-1}{3n^4-2n^5+3}$; h) $\lim_{n\to\infty}\frac{2n^4-4n^6+3n^5-1}{3n^5-2n^4+3}$;

3. Calculaţi:

a) $\lim_{n\to\infty}\left(\frac{4n^3-1}{2n^3+n^2}\right)^3$; b) $\lim_{n\to\infty}\left(\frac{6n^3-1}{2n^3+n^2}\right)^2$; c) $\lim_{n\to\infty}\left(\frac{8n^3-7}{4n^3+2n^2}\right)^3$; d) $\lim_{n\to\infty}\left(\frac{9n^3-7}{3n^3+2n^2}\right)^2$;

e) $\lim_{n\to\infty}\left(\frac{5n^3-1}{2n^3+n^2}\right)^{\frac{n^2-1}{n^2+1}}$; f) $\lim_{n\to\infty}\left(\frac{2n^3+n^2}{5n^3-1}\right)^{\frac{1-n^2}{n^2+1}}$; g) $\lim_{n\to\infty}\left(\frac{4n^2-5}{25n^2-4n}\right)^{\frac{4-n}{2n-5}}$; h) $\lim_{n\to\infty}\left(\frac{25n^2-4n}{4n^2-5}\right)^{\frac{n-4}{2n-5}}$;

i) $\lim_{n\to\infty}\left(\frac{2n^2-n}{5n^2-3}\right)^{\frac{1-2n^3}{n^3+2}}$; j) $\lim_{n\to\infty}\left(\frac{5n^2-3}{2n^2-n}\right)^{\frac{2n^3-1}{n^3+2}}$; k) $\lim_{n\to\infty}\left(\frac{2n^3-2n^4}{5n^4-3}\right)^{\frac{1-2n^2}{n^2+5}}$; l) $\lim_{n\to\infty}\left(\frac{5n^4-3}{2n^3-2n^4}\right)^{\frac{1+2n^2}{n^2-5}}$.

4. Calculaţi:

a) $\lim_{n\to\infty}\left(\frac{4n^2-3n}{4n^2+2n+1}\right)^{5n+2}$; b) $\lim_{n\to\infty}\left(\frac{3n^2-2n}{3n^2+3n+1}\right)^{5n+3}$; c) $\lim_{n\to\infty}\left(\frac{5n^2-2n}{5n^2+3n+1}\right)^{5n+1}$; d) $\lim_{n\to\infty}\left(\frac{n^2-2n}{n^2+3n+1}\right)^{5n+4}$;

e) $\lim_{n\to\infty}\left(\frac{8n^6-2n^4-1}{6n^8+5n^4+1}\right)^{7n^2+1}$; f) $\lim_{n\to\infty}\left(\frac{6n^8-5n^4-1}{6n^8+2n^4+1}\right)^{7n^4+1}$; g) $\lim_{n\to\infty}\left(\frac{7n^5-4n^3-1}{7n^5+2n^3+1}\right)^{6n^2+1}$; h) $\lim_{n\to\infty}\left(\frac{5n^7-2n^3-1}{5n^7+4n^3+1}\right)^{6n^4+1}$;

i) $\lim_{n\to\infty}\left(\frac{5n^6-2n^3-1}{5n^6+n^3+1}\right)^{3n^3+1}$; j) $\lim_{n\to\infty}\left(\frac{6n^5-n^3-1}{6n^5+2n^3+1}\right)^{3n^2+1}$; k) $\lim_{n\to\infty}\left(\frac{6n^7-2n^4-1}{6n^7+3n^4+1}\right)^{5n^3+1}$; l) $\lim_{n\to\infty}\left(\frac{7n^6-3n^4-1}{7n^6+2n^4+1}\right)^{5n^2+2}$.

5. Calculaţi:

a) $\lim_{n\to\infty}(5n^2+4)\sin\left(\frac{4n^3+5}{2n^5+7n}\right)$; b) $\lim_{n\to\infty}(4n^3+5)\sin\left(\frac{5n^2+4}{2n^5+7n}\right)$; c) $\lim_{n\to\infty}(3n^2+1)\sin\left(\frac{2n^2+3}{7n^4+4n}\right)$; d) $\lim_{n\to\infty}(2n^2+3)\sin\left(\frac{3n^2+1}{7n^4+4n}\right)$;

e) $\lim_{n\to\infty}(5n^3+1)\sin\left(\frac{2n^2+7}{8n^5+3n}\right)$; f) $\lim_{n\to\infty}(2n^2+7)\sin\left(\frac{5n^3+1}{8n^5+3n}\right)$; g) $\lim_{n\to\infty}(3n^2+4)\sin\left(\frac{5n^4+3}{7n^6+5n}\right)$; h) $\lim_{n\to\infty}(5n^4+3)\sin\left(\frac{3n^2+4}{7n^6+5n}\right)$;

i) $\lim_{n\to\infty}(4n^2+3)\sin\left(\frac{5n^2-1}{5n^5+6n}\right)$; j) $\lim_{n\to\infty}(3n^2+4)\sin\left(\frac{5n^2-1}{6n^5+5n}\right)$; k) $\lim_{n\to\infty}(5n^3-1)\sin\left(\frac{4n^3+3}{6n^5+5n}\right)$; l) $\lim_{n\to\infty}(5n^3-1)\sin\left(\frac{3n^3+4}{5n^5+6n}\right)$.

6. Calculaţi $\lim\limits_{n\to\infty} \sqrt[n]{x_n}$, ştiind că, pentru orice $n \geq 1$, avem:

a) $x_n = \left(\dfrac{5n-6}{7n-1}\right)^{3n}$;

b) $x_n = \left(\dfrac{4n-7}{7n-3}\right)^{5n}$;

c) $x_n = \left(\dfrac{3n-2}{5n-3}\right)^{2n}$;

d) $x_n = \left(\dfrac{2n-7}{7n-2}\right)^{3n}$;

e) $x_n = \left(\dfrac{4n^2-5n+3}{4n^2-3n+1}\right)^{n^2}$;

f) $x_n = \left(\dfrac{4n^2-3n+3}{4n^2-5n+1}\right)^{n^2}$;

g) $x_n = \left(\dfrac{5n^2-3n+4}{5n^2-2n+3}\right)^{n^2}$;

h) $x_n = \left(\dfrac{5n^2-2n+4}{5n^2-3n+3}\right)^{n^2}$;

i) $x_n = \left(\dfrac{2n^2-4n+5}{2n^2-3n+6}\right)^{n^2}$;

j) $x_n = \left(\dfrac{2n^2-3n+5}{2n^2-4n+6}\right)^{n^2}$;

k) $x_n = \left(\dfrac{3n^3-4n^2+1}{3n^3-2n+2}\right)^{n^3}$;

l) $x_n = \left(\dfrac{3n^3-2n+1}{3n^3-4n^2+2}\right)^{n^3}$.

7. Calculaţi $\lim\limits_{n\to\infty} \dfrac{x_{n+1}}{x_n}$, ştiind că, pentru orice $n \geq 1$, avem:

a) $x_n = \dfrac{n^3}{n! \cdot 4^n}$; b) $x_n = \dfrac{n^4}{n! \cdot 3^n}$; c) $x_n = \dfrac{n^4}{n! \cdot 5^n}$; d) $x_n = \dfrac{n^5}{n! \cdot 4^n}$.

8. Calculaţi $\lim\limits_{n\to\infty} \dfrac{x_{n+1}}{x_n}$, ştiind că, pentru orice $n \geq 1$, avem:

a) $x_n = \dfrac{n! \cdot 3^n}{7 \cdot 11 \cdot 15 \cdot \ldots \cdot (4n+3)}$;

b) $x_n = \dfrac{n! \cdot 5^n}{5 \cdot 9 \cdot 13 \cdot \ldots \cdot (4n+1)}$;

c) $x_n = \dfrac{n! \cdot 4^n}{8 \cdot 13 \cdot 18 \cdot \ldots \cdot (5n+3)}$;

d) $x_n = \dfrac{n! \cdot 3^n}{7 \cdot 12 \cdot 17 \cdot \ldots \cdot (5n+2)}$.

9. Calculaţi $\lim\limits_{n\to\infty} \dfrac{x_{n+1}}{x_n}$ şi $\lim\limits_{n\to\infty} n \cdot \left(\dfrac{x_n}{x_{n+1}} - 1\right)$, ştiind că, pentru orice $n \geq 1$, avem:

a) $x_n = \dfrac{n! \cdot 4^n}{7 \cdot 11 \cdot 15 \cdot \ldots \cdot (4n+3)}$;

b) $x_n = \dfrac{n! \cdot 4^n}{5 \cdot 9 \cdot 13 \cdot \ldots \cdot (4n+1)}$;

c) $x_n = \dfrac{n! \cdot 5^n}{8 \cdot 13 \cdot 18 \cdot \ldots \cdot (5n+3)}$;

d) $x_n = \dfrac{n! \cdot 5^n}{7 \cdot 12 \cdot 17 \cdot \ldots \cdot (5n+2)}$.

10. Fie $(a_n)_{n\geq 1}$ şi $(b_n)_{n\geq 1}$ două şiruri de numere reale. Calculaţi $\lim\limits_{n\to\infty} \dfrac{a_{n+1}-a_n}{b_{n+1}-b_n}$ ştiind că, pentru orice $n \geq 1$, avem:

a) $a_n = 1^4 + 2^4 + 3^4 + \ldots + n^4$ şi $b_n = 4n^5$;

b) $a_n = 1^6 + 2^6 + 3^6 + \ldots + n^6$ şi $b_n = 6n^7$.