

Probleme propuse:

1. Să se calculeze:

$$\begin{aligned} \text{a)} \quad & \lim_{n \rightarrow \infty} \frac{4n^6 + 5n^5 + 6n^4 + 2}{4n^4 + 5n^5 + 6n^6 + 3} \\ \text{b)} \quad & \lim_{n \rightarrow \infty} \frac{4n^7 + 5n^8 + 6n^3 + 4}{2n^3 + 7n^6 + 6n^8 + 5} \end{aligned}$$

$$\begin{aligned} \text{c)} \quad & \lim_{n \rightarrow \infty} \frac{2n^2 + 3n^3 + 4n^4 + 3}{5n^4 + 6n^2 + 3n^3 + 1} \\ \text{d)} \quad & \lim_{n \rightarrow \infty} \frac{3n^8 + 4n^7 + 5n^6 + 5}{5n^6 + 6n^7 + 7n^8 + 8} \end{aligned}$$

2. Să se calculeze:

$$\begin{aligned} \text{a)} \quad & \lim_{n \rightarrow \infty} \left(\frac{n^2 + 3}{n^3 + 2} \right)^{6n} \\ \text{b)} \quad & \lim_{n \rightarrow \infty} \left(\frac{n^3 + 2}{n^2 + 3} \right)^{6n} \\ \text{c)} \quad & \lim_{n \rightarrow \infty} \left(\frac{n^3 + 3n^2 + 1}{n^2 + 3n^3 + 1} \right)^{n^2 - 1} \end{aligned}$$

$$\begin{aligned} \text{d)} \quad & \lim_{n \rightarrow \infty} \left(\frac{n^7 + 6n^5 - 3n^8 + 1}{4n^8 + 2n^7 - 5n^5 + 2} \right)^{n^3} \\ \text{e)} \quad & \lim_{n \rightarrow \infty} \left(\frac{3n^2 - 4n^3 - 5n^6 + 1}{2n^6 - 3n^5 + 4n^4 - 2} \right)^{n^2} \\ \text{f)} \quad & \lim_{n \rightarrow \infty} \left(\frac{3n^2 - 4n^3 - 5n^6 + 1}{2n^6 - 3n^5 + 4n^4 - 2} \right)^{1 - n^2} \end{aligned}$$

3. Să se calculeze:

$$\begin{aligned} \text{a)} \quad & \lim_{x \rightarrow -\infty} \frac{1 - x^4}{1 + x^3} \\ \text{b)} \quad & \lim_{x \rightarrow -\infty} \frac{1 - x^4}{1 - x^3} \\ \text{c)} \quad & \lim_{x \rightarrow -\infty} \frac{1 - x^3}{1 + x^4} \\ \text{d)} \quad & \lim_{x \rightarrow -\infty} \frac{1 - x^3}{1 - x^4} \end{aligned}$$

$$\begin{aligned} \text{e)} \quad & \lim_{x \rightarrow -\infty} \frac{1 - x^5}{1 + x^3} \\ \text{f)} \quad & \lim_{x \rightarrow -\infty} \frac{1 - x^5}{1 - x^3} \\ \text{g)} \quad & \lim_{x \rightarrow -\infty} \frac{1 - x^3}{1 + x^5} \\ \text{h)} \quad & \lim_{x \rightarrow -\infty} \frac{1 - x^3}{1 - x^5} \end{aligned}$$

$$\begin{aligned} \text{i)} \quad & \lim_{x \rightarrow -\infty} \frac{3 - 2x^5}{2 - 3x^5} \\ \text{j)} \quad & \lim_{x \rightarrow -\infty} \frac{3 - 2x^5}{2 + 3x^5} \\ \text{k)} \quad & \lim_{x \rightarrow -\infty} \frac{3 + 2x^5}{2 - 3x^4} \\ \text{l)} \quad & \lim_{x \rightarrow -\infty} \frac{3 - 2x^5}{2 - 3x^4} \end{aligned}$$

4. Să se calculeze:

$$\begin{aligned} \text{a)} \quad & \lim_{x \searrow (-1)} \frac{2 + x}{1 + x} \\ \text{b)} \quad & \lim_{x \searrow (-2)} \frac{3 - x}{2 + x} \end{aligned}$$

$$\begin{aligned} \text{c)} \quad & \lim_{x \searrow (-3)} \frac{2 + x^2}{3 + x} \\ \text{d)} \quad & \lim_{x \nearrow (-1)} \frac{2 - x^2}{1 + x} \end{aligned}$$

$$\begin{aligned} \text{e)} \quad & \lim_{x \nearrow (-2)} \frac{3 - x^2}{2 + x} \\ \text{f)} \quad & \lim_{x \nearrow (-3)} \frac{x^2 - x}{3 + x} \end{aligned}$$

5. Să se calculeze:

$$\begin{aligned} \text{a)} \quad & \lim_{x \rightarrow \infty} \left(\frac{3x + 2}{3x + 5} \right)^x \\ \text{b)} \quad & \lim_{x \rightarrow \infty} \left(\frac{3x + 5}{3x + 2} \right)^x \\ \text{c)} \quad & \lim_{x \rightarrow \infty} \left(\frac{3x^2 + 2x + 1}{3x^2 + 5x + 3} \right)^{5x} \\ \text{d)} \quad & \lim_{x \rightarrow \infty} \left(\frac{3x^2 + 5x + 3}{3x^2 + 2x + 1} \right)^{5x} \end{aligned}$$

$$\begin{aligned} \text{e)} \quad & \lim_{x \rightarrow \infty} \left(\frac{3x^2 - 2x + 1}{3x^2 + 5x - 3} \right)^{4x} \\ \text{f)} \quad & \lim_{x \rightarrow \infty} \left(\frac{3x^2 + 5x - 3}{3x^2 - 2x + 1} \right)^{4x} \\ \text{g)} \quad & \lim_{x \rightarrow \infty} \left(\frac{3x^3 + 2x + 1}{3x^3 + 5x + 3} \right)^{5x} \end{aligned}$$

$$\begin{aligned} \text{h)} \quad & \lim_{x \rightarrow \infty} \left(\frac{3x^3 + 5x + 3}{3x^3 + 2x + 1} \right)^{5x^2} \\ \text{i)} \quad & \lim_{x \rightarrow \infty} \left(\frac{3x^4 + 2x + 1}{3x^4 + 5x + 3} \right)^{5x^4} \\ \text{j)} \quad & \lim_{x \rightarrow \infty} \left(\frac{3x^4 + 5x + 3}{3x^4 + 2x + 1} \right)^{5x^4} \end{aligned}$$

6. Să se calculeze:

$$\begin{aligned} \text{a)} \quad & \lim_{x \rightarrow 6} \frac{\sin(x - 6)}{5x - 30} \\ \text{b)} \quad & \lim_{x \rightarrow 6} \frac{\sin(6 - x)}{5x - 30} \end{aligned}$$

$$\begin{aligned} \text{c)} \quad & \lim_{x \rightarrow 6} \frac{\sin(x^2 - 36)}{5x - 30} \\ \text{d)} \quad & \lim_{x \rightarrow 6} \frac{\sin(36 - x^2)}{5x - 30} \end{aligned}$$

$$\begin{aligned} \text{e)} \quad & \lim_{x \rightarrow 6} \frac{\sin(5x - 30)}{x^2 - 36} \\ \text{f)} \quad & \lim_{x \rightarrow 6} \frac{\sin(30 - 5x)}{36 - x^2} \end{aligned}$$

7. Să se calculeze:

a) $\lim_{x \rightarrow 7} \frac{\ln(1 + (x - 7))}{3x - 21}$

c) $\lim_{x \rightarrow 6} \frac{\ln(x - 5)}{36 - x^2}$

e) $\lim_{x \rightarrow 6} \frac{\ln(x^2 - 35)}{4x - 24}$

b) $\lim_{x \rightarrow 7} \frac{\ln(1 + (x - 7))}{21 - 3x}$

d) $\lim_{x \rightarrow 6} \frac{\ln(x - 5)}{x^2 - 36}$

f) $\lim_{x \rightarrow 6} \frac{\ln(37 - x^2)}{24 - 4x}$

8. Să se calculeze:

(a) $\lim_{x \rightarrow 3} \frac{5^x - 125}{3x - 15}$

(c) $\lim_{x \rightarrow 3} \frac{4^x - 64}{3x - 15}$

(e) $\lim_{x \rightarrow 2} \frac{2 - \sqrt{2}^x}{4 - x^2}$

(b) $\lim_{x \rightarrow 3} \frac{5^x - 125}{15 - 3x}$

(d) $\lim_{x \rightarrow 3} \frac{216 - 6^x}{36 - x^2}$

(f) $\lim_{x \rightarrow 2} \frac{3 - \sqrt{3}^x}{x^2 - 9}$

9. Să se studieze natura seriei:

a) $\sum_{n=1}^{\infty} \left(\frac{n^2 - 3n + 2}{4n^2 + 3n + 2} \right)^{2n}$

c) $\sum_{n=1}^{\infty} \left(\frac{4n^3 - 4n + 2}{5n^3 + 3n^2 + 2} \right)^{3n}$

e) $\sum_{n=1}^{\infty} \left(\frac{5n^5 - 4n^2 + 1}{2n^5 + 3n^3 - 2} \right)^{n^2}$

b) $\sum_{n=1}^{\infty} \left(\frac{4n^2 - 3n + 2}{3n^2 + 3n + 2} \right)^{4n}$

d) $\sum_{n=1}^{\infty} \left(\frac{5n^5 - 4n^2 + 1}{2n^5 + 3n^3 - 2} \right)^{3n}$

f) $\sum_{n=1}^{\infty} \left(\frac{3n^6 - 4n^5 + 1}{4n^6 + 3n^3 - 2} \right)^{n^2}$

10. Să se studieze natura seriei:

a) $\sum_{n=1}^{\infty} \frac{3n + 7}{n!}$

c) $\sum_{n=1}^{\infty} \frac{3n^2 + 7n + 1}{n! \cdot 2^n}$

e) $\sum_{n=1}^{\infty} \frac{5^n \cdot n!}{n^3}$

b) $\sum_{n=1}^{\infty} \frac{2^n}{n!}$

d) $\sum_{n=1}^{\infty} \frac{3^n}{n! \cdot n^2}$

f) $\sum_{n=1}^{\infty} \frac{2^n \cdot n^2}{n!}$

11. Să se calculeze derivata de ordinul întâi a funcției $f : (0, +\infty) \rightarrow \mathbb{R}$ definită prin:

a) $f(t) = 3t^2 - 2t + 1 - \sin(t)$

c) $f(t) = 5t^4 - 3t^2 + 2t + 2^t$

e) $f(t) = 3t^7 - 4t^8 - 3t^3 + \ln(t)$

b) $f(t) = 4t^3 - 2t^2 + 5 - \cos(t)$

d) $f(t) = 2t^5 - 3t^6 - 3t^2 + e^t$

f) $f(t) = 6t^5 - 4t^{12} + 5t^2 + \sqrt{t}$

12. Să se calculeze derivata de ordinul întâi a funcției $f : (0, +\infty) \rightarrow \mathbb{R}$ definită prin:

a) $f(t) = \frac{5t + 3}{3t + 5}$

c) $f(t) = \frac{3t + 2}{2t + 3}$

e) $f(t) = \frac{3t + 2}{2t^2 + 3}$

b) $f(t) = \frac{3t + 2}{2t + 3}$

d) $f(t) = \frac{3t^2 + 2}{2t + 3}$

f) $f(t) = \frac{3t^3 + 2}{2t^4 + 3}$

13. Să se calculeze derivata de ordinul întâi a funcției $f : (0, +\infty) \rightarrow \mathbb{R}$ definită prin:

a) $f(t) = t^2 \cdot \sin(t)$

c) $f(t) = t^4 \cdot \ln(t)$

e) $f(t) = t^6 \cdot \arctg(t)$

b) $f(t) = t^3 \cdot \cos(t)$

d) $f(t) = t^5 \cdot 5^t$

f) $f(t) = t^7 \cdot \arcsin(t)$

14. Să se calculeze derivata de ordinul întâi a funcției $f : (0, +\infty) \rightarrow \mathbb{R}$ definită prin:

a) $f(t) = \ln(\sin(5t) + 5^t + 1)$

c) $f(t) = \ln(\sin(t + 2) + e^t)$

e) $f(t) = \cos(\ln(t) + 2^t)$

b) $f(t) = \ln(\cos(3t) + 3^t + 1)$

d) $f(t) = \sin(\ln(t) + e^t)$

f) $f(t) = \sin(\cos(t) + 3^t)$