Mathematical Society
of South Eastern Europe

Technical University
"Gheorghe Asachi" of Iasi

Romanian
Mathematical Society

South Eastern European Mathematical Olympiad for University Students

Iaşi, Romania - April 11, 2024

Problem 1. Let $(x_n)_{n\geq 1}$ be the sequence defined by $x_1 \in (0,1)$ and $x_{n+1} = x_n - \dfrac{x_n^2}{\sqrt{n}}$ for all $n \geq 1$.

Find the values of $\alpha \in \mathbb{R}$ for which the series $\displaystyle\sum_{n=1}^{\infty} x_n^{\alpha}$ is convergent.

Problem 2. Let $A, B \in \mathcal{M}_n(\mathbb{R})$ two real, symmetric matrices with nonnegative eigenvalues. Prove that $A^3 + B^3 = (A+B)^3$ if and only if $AB = \mathcal{O}_n$.

Problem 3. For every $n \geq 1$ define x_n by

$$x_n = \int_0^1 \ln(1 + x + x^2 + \ldots + x^n) \cdot \ln \frac{1}{1-x}\, \mathrm{d}x, \quad n \geq 1.$$

a) Show that x_n is finite for every $n \geq 1$ and $\lim\limits_{n\to\infty} x_n = 2$.

b) Calculate $\lim\limits_{n\to\infty} \dfrac{n}{\ln n}(2 - x_n)$.

Problem 4. Let $n \in \mathbb{N}$, $n \geq 2$. Find all the values $k \in \mathbb{N}$, $k \geq 1$, for which the following statement holds:

"If $A \in \mathcal{M}_n(\mathbb{C})$ is such that $A^k A^* = A$, then $A = A^*$."

(here, $A^* = \overline{A}^t$ denotes the transpose conjugate of A).

SEEMOUS 2024
APRIL 9-14, IAŞI, ROMÂNIA