1. Let $A = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$.

   (a) Find the eigenvalues and eigenspaces of A.

   (b) Prove that A can be brought to a diagonal form and find its diagonal form.

   (c) Find expressions for A^n and A^{-1}.

   (d) Which are the eigenvalues of A^{-1}? What about $2A - 3I_3$?

Grading: 4p (2.5p+0.5p+0.5p+0.5p)

2. Orthonormalize

$$S = \{v_1 = (1,0,1),\ v_2 = (1,0,-1),\ v_3 = (0,3,4)\}$$

   by means of the Gram-Schmidt procedure.

Grading: 2p

3. (a) Is

$$f : \mathbb{R}^3 \to \mathbb{R}, \quad f(x_1, x_2, x_3) = x_1x_2 - x_2x_3,$$

   a linear form?

   (b) Consider the quadratic form

$$h : \mathbb{R}^3 \to \mathbb{R}, \quad h(x_1, x_2, x_3) = x_1^2 - x_3^2 - 4x_1x_2 + 4x_2x_3.$$

   i. Find the associated bilinear form and determine its rank.

   ii. Find the canonical form of h (basis not requested) and its signature. Is h degenerate, or nondegenerate?

   iii. Determine whether or not h is positive definite. Is there any $(x_1, x_2, x_3) \in \mathbb{R}^3$ such that $h(x_1, x_2, x_3) < 0$?

Grading: 3p (0.5p+2.5p (1p+1p+0.5p))

Each problem has its own grading system. The final grade is the sum of the grades received for each problem+1p for showing up. Allotted time: 90 minutes.